# COMPREHENSIVE EXAMINATION

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Course No.: BITS F472

DATE: Dec. $8^{th}$ 2015 (AN)        MAX. TIME: 3 hrs.

**Q. 1.** Effective isotropic radiated power (EIRP) is an important quantity in satellite link budget analysis.

(a). Express EIRP of an earth station's transmitter (TX) in terms of transmit power $P_T$, aperture efficiency $\eta$, effective aperture area $A_e$, and operating frequency $f$.

(b). From (a), write down expression for EIRP in $dB$.

(c). Plot EIRP in $dB$ as a function of $10\log_{10} f$, assuming all other quantities fixed. What is the slope of this curve? Use this curve's nature to comment on the behavior of EIRP with operating wavelength $\lambda$.

(d). An earth station uses a 1 $KW$ high power amplifier (HPA). Further, it also uses 20 m Cassegrain antenna with gain 65 $dB$ at a free space wavelength of 2.1 cm. If the loss in the waveguide that connects HPA to the feed is 1 dBm, compute the earth station EIRP. **[1 + 1 + 3 + 2 points]**

**Q. 2.** A geosynchronous satellite is located at a distance of 36000 KM from the earth station whose receiver has bandwidth of 10 MHz.

(a). Carry out downlink budget analysis at 12 GHz using the following data and compute downlink carrier power-to-noise power ratio (CNR) in $dB$. In your computations, use Boltzmann's constant value as $1.38 \times 10^{-23}$ $J/^{\circ}K$.

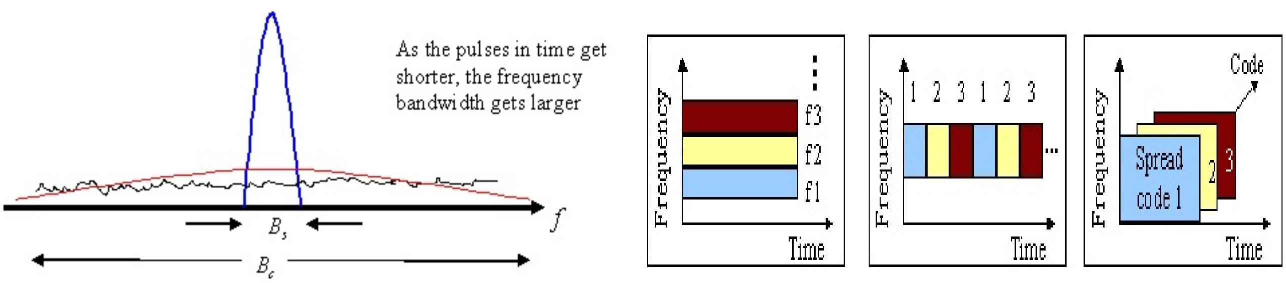
(b). What happens to CNR of the downlink if the transmit power at satellite is halved and the bandwidth at the earth station is doubled?. Compute the resultant CNR and % variation. **[4 + 3 points]**

| At satellite | At earth station |
|---|---|
| Transmitting power = 40 $dBm$ | Dish antenna diameter = 10 m |
| Antenna aperture efficiency = 65% | Antenna aperture efficiency = 65% |
| Dish antenna diameter = 0.5 m | Equivalent noise temperature = 300$^{\circ}K$ |

**Q. 3.** A satellite receiver (RX) system comprises of, in sequence, antenna, low noise amplifier (LNA), cable, and satellite receiver. The antenna noise temperature $T_a$ is 35$^\circ K$. The gain of LNA, denoted by G, is 50 dB and its noise temperature $T_g$ is 150$^\circ K$. Further, the cable loss $L$ is 5 $dB$ and the receiver noise figure $F$ is 12 $dB$. Assume reference temperature $T_0 = 290^\circ K$.

Draw a clear block diagram of the satellite RX system indicating all the quantities. Write down expression for overall noise temperature of the satellite receiver system in term of $T_a$, $T_g$, $G$, $L$, $F$, and $T_0$. Compute it's value using the given data. **[1 + 2 + 2 points]**

**Q. 4.** Briefly describe the following illustrations. **[1 + 1 points]**




(a) **?.** (b) **?.**

Fig. 1: Pertaining to Q. No. (4).

**Q. 5.** Consider a Gaussian random variable (RV) $Y$ with mean $-2.2724$ and standard deviation $1.6469$.

(a). Compute mean and standard deviation of $Z = \exp(Y)$.

(b). Consider a 100% shadowed mobile satellite (MSAT) path characterized by the distribution of $Z$ in (a). Compute the percentage of time the signal amplitude will fade below 9 $dB$. **[2 + 3 points]**

**Q. 6.** Every subsystem of earth station except antenna employs some form of redundancy to enhance reliability. Assuming that failures occur randomly, the probability of the hardware being operated larger than the time interval is given by the exponential distribution as $R(t) = \exp(-\lambda t)$, where $\lambda$ is the average failure rate. Mean time to failure (MTTF), denoted by $\mu$, is given by

$$\mu = \int_0^\infty -t\, dR(t).$$

(a). Determine $\mu$ in terms of $\lambda$.

(b). If 'n' independent subsystems are connected in series, show that, the overall MTTF $\mu_s = \frac{1}{\sum_{i=1}^{n} \lambda_i}$, where $\lambda_i$ is the average failure rate of each subsystem.

(c) When ‘n’ identical subsystems are connected in parallel, derive expression for the overall MTTF $\mu_p$ in integral form. **[2 + 2 + 2 points]**

**Q. 7.** (a). Describe the following in just ONE meaningful and complete sentence.

i). Pre-emphasis and de-emphasis in frequency modulation (FM) ii). Processing gain in direct sequence spread spectrum iii). Jamming in code division multiple access iv). Carson’s rule

b). Show the following by suitable drawing: i). Simplified global positioning system (GPS) RX. ii). Differential GPS. **[2 + 2 + 1 points]**

**Q. 8.** (a). Consider an AWGN channel. Let $E_b$ denote bit energy, $\frac{\mathcal{C}}{B}$ denote bandwidth-normalized capacity, and $N_0$ denote power spectral density. Write down Shannon-Hartley law in terms of $\frac{E_b}{N_0}$, and bandwidth-normalized capacity.

(b). Find the asymptotic limit of $\frac{E_b}{N_0}$ in $dB$ as bandwidth $\rightarrow \infty$. **[1 + 2 points]**

## I. Answers

**Q. 1.** (a). We know that EIRP = $P_T G_T$. Since $G_T = \eta \frac{4\pi A_e f^2}{c^2}$, EIRP can be expressed as

$$EIRP = \frac{4\pi}{c^2} \eta P_T A_e f^2,$$

where ‘c’ is velocity of light.

(b). EIRP (dB) can be expressed as

$$EIRP(dB) = 10\log_{10}\frac{4\pi}{c^2} + 10\log_{10}\eta + 10\log_{10}P_T + 10\log_{10}A_e + 20\log_{10}f.$$

(c). Given that all other quantities, except frequency, are fixed. Therefore, we have

$$EIRP(dB) = K_{dB} + 2 \times 10\log_{10}f,$$

where $K_{dB} = 10\log_{10}\frac{4\pi}{c^2} + 10\log_{10}\eta + 10\log_{10}P_T + 10\log_{10}A_e$.

It is clear that, EIRP (dB) varies (increases) linearly with $10\log_{10}f$. Slope of the curve is two.

Since frequency is inversely proportional to wavelength, EIRP (dB) decreases linearly with the increase in $\lambda$.

(d). EIRP without accounting loss = $125\ dBm$. After accounting $1\ dBm$ loss, EIRP = $124\ dBm$ or $94$ dB.

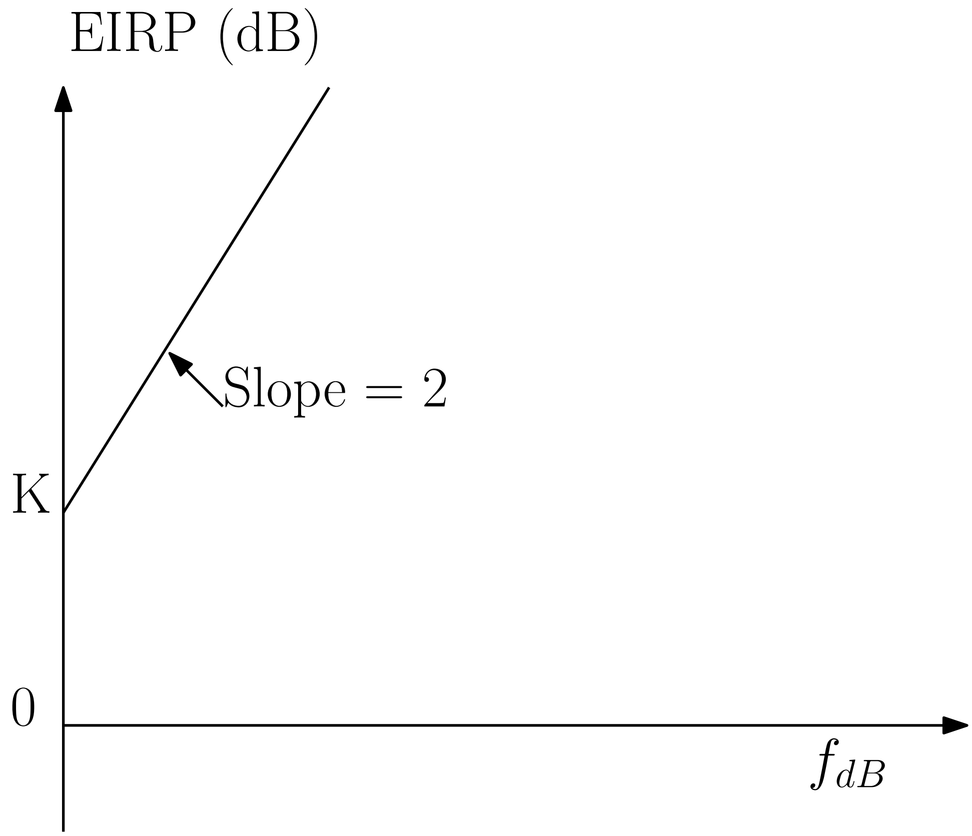



Fig. 2: Q. 1. (c). Illustrating linear relationship between EIRP (dB) and $f_{dB} = 10\log_{10} f$.

**Q. 2.** (a). CNR of the downlink = $32.89\ dB$.

(b). Resultant CNR = $32.89\ dB$ - $6\ dB$ = $26.87\ dB$. In $dB$ scale, % variation = $18.3\%$. In linear scale, % variation = $75.0\%$.

**Q. 3.** Block diagram: This comprises of antenna, LNA, RX, and, cable that connects LNA and RX as shown in figure 3.

The system's overall noise temperature = $T_a + T_g + \frac{(L-1)T_0}{G} + \frac{L(F-1)T_0}{G}$.

Using the data, we see that the system's overall noise temperature is $185.15°K$.

**Q. 4.** (a). This figure illustrates spreading of a narrow-bandwidth data signal to a very large bandwidth using PN sequence. This is called spread spectrum modulation which is used in satellite systems, such as GPS. The ratio of the two bandwidths, $\frac{B_c}{B_s}, B_c >> B_s$, gives the processing gain. The figure also shows the noise/interference signal's power spectral density (PSD) that was also spread over large bandwidth.

(b). This figure illustrates three basic multiple access schemes, namely, time division multiple access, frequency division multiple access, and, code division multiple access (CDMA). These are most popular

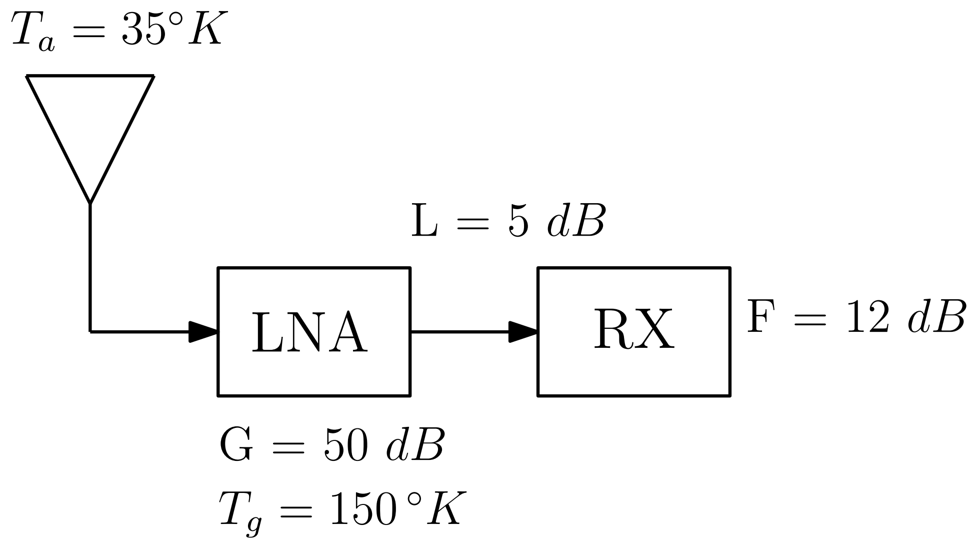



Fig. 3: Block diagram pertaining to Q. No. 3.

multiple access schemes used in satellite communication systems. For example, CDMA is used in mobile satellite (MSAT) systems and GPS.

**Q. 5.** (a). Mean: $\mu_Z = \exp\left(\mu_Y + \frac{{\sigma_Y}^2}{2}\right) = 0.4$.

Variance: ${\sigma_Z}^2 = \exp\left(2\mu_Y + 2{\sigma_Y}^2\right) - {\mu_Z}^2 = 2.25$. Therefore, standard deviation is $1.5$.

(b). The desired probability is given by

$$\mathcal{P}(Z < 10^{0.9}) = \frac{1}{\sqrt{2\pi}1.5}\int_0^{7.9433} \frac{1}{z}\exp\left(-\frac{(\ln z - 0.4)^2}{4.5}\right) dz. \quad (1)$$

Use substitution method to simplify the definite integral. This approach yields probability $1-Q\left(\frac{\ln(7.9433)-0.4}{1.5}\right)$. Simplifying further yields 0.8676.

Alternatively, using Wolframalphas's definite-integral calculator[1], we get desired probability 0.8676 or 86.76%.

**Q. 6.** (a). From the given MTTF formula, we have

$$\mu = \lambda\int_0^{\infty} t\exp(-\lambda t)\,dt.$$

Using integration by parts, it is easy to show that $MTTF = \frac{1}{\lambda}$.

(b). Since subsystems are independent, we have $R_s(t) = \exp\left(-\left(\lambda_1 + \ldots + \lambda_n\right)t\right)$. Let $\lambda_s = \lambda_1 + \ldots + \lambda_n$. Using the result in (a), we can show that $\mu_s = \frac{1}{\sum_{i=1}^{n}\lambda_i}$.

[1]http://www.wolframalpha.com/widgets/view.jsp?id=8ab70731b1553f17c11a3bbc87e0b605

(c). In this case, we have $R_p(t) = 1 - (1 - \exp(-\lambda_1 t))(1 - \exp(-\lambda_2 t)) \ldots (1 - \exp(-\lambda_n t))$. Since the 'n' subsystems are identical, $R_p(t) = 1 - (1 - \exp(-\lambda t))^n$.

Therefore, $\mu_p$, in integral form, is given by

$$\mu = n\lambda \int_0^{\infty} t \exp(-\lambda t) \left(1 - \exp(-\lambda t)\right)^{n-1} dt.$$

**Q. 7.** (a). i). In frequency modulation (FM), pre-emphasis, which is performed before modulator, boosts high frequency signal components, and, de-emphasis, which is performed after demodulator, reduce excessive noise at high frequencies so as to improve SNR.

ii). Processing gain in direct sequence spread spectrum is the ratio of chip rate to the data rate or the ratio of data bit duration to the chip duration.

iii). Jamming in code division multiple access is the intentional interference (undesired signal power) whose effect could be much more dominative than noise.

iv). Carson's rule is an empirical rule to compute bandwidth of a wideband FM signal, which is given by twice to the'sum of peak frequency deviation and maximum frequency component'.

(b). i). Simplified block diagram of GPS (C/A code):

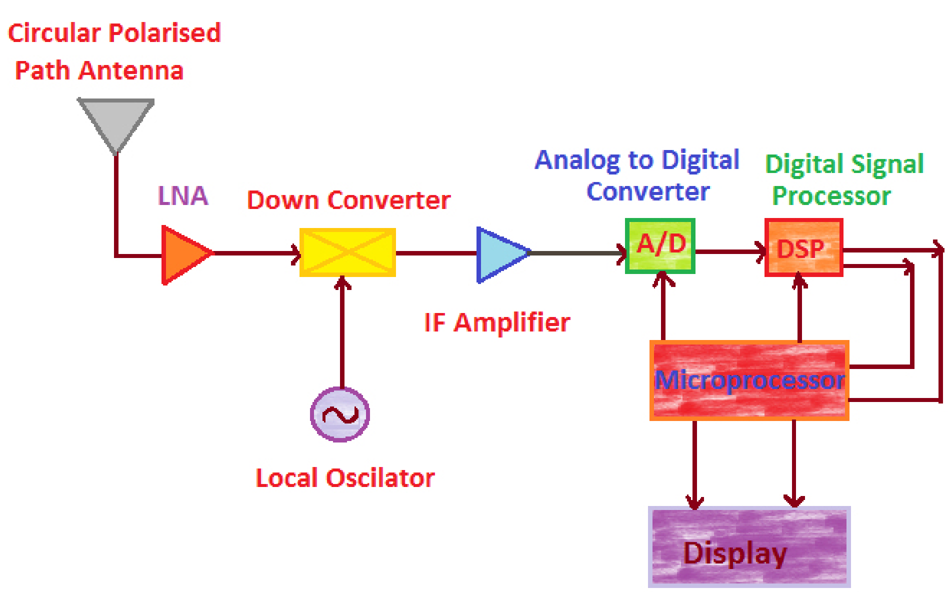



Fig. 4: Q. 7. (b). (i). Simplified block diagram of C/A code GPS.

**Q. 8.** (a). Let $T_b$ be the bit duration. The transmission bit-rate is given by $R_b = \frac{1}{T_b}$. Signal power at the RX is $E_b R_b$ and noise power is $N_0 B$, where B is the RX bandwidth. For AWGN channel, capacity

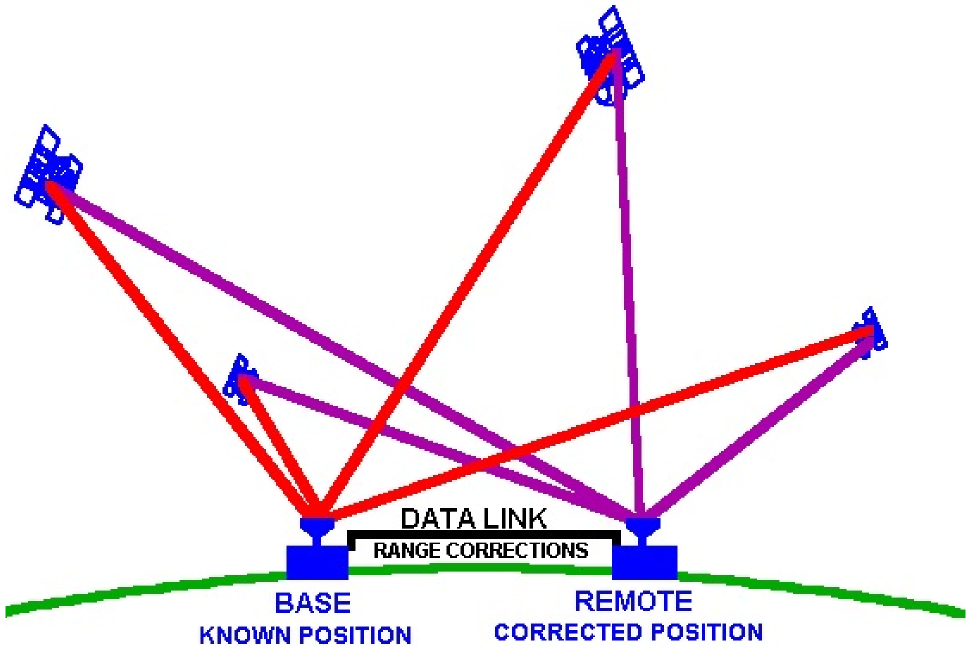



Fig. 5: Q. 7. (b). (ii): Illustration of differential GPS.

$\mathcal{C}$ is given by $\mathcal{C} = B\log_2(1 + SNR)$, where $SNR = \frac{E_b R_b}{N_0 B}$. Therefore, we can express one of the most celebrated law in information theory, Shannon-Hartley law, as $\frac{\mathcal{C}}{B} = \log_2(1 + \frac{E_b R_b}{N_0 B})$ bps/Hz.

(b). Assume that the transmission bit rate is equal to the channel capacity. From Shannon-Hartley's law expressed in (a), we have

$$\frac{E_b}{N_0} = \frac{2^{\frac{\mathcal{C}}{B}} - 1}{\frac{\mathcal{C}}{B}}.$$

As bandwidth $\to \infty$, we have $\frac{\mathcal{C}}{B} \to 0$. The asymptotic limit of $\frac{E_b}{N_0}$ as $B \to \infty$ is given by[2]

$$\lim_{B\to\infty} \frac{E_b}{N_0} = \ln 2.$$

In dB scale, the asymptotic limit of bit energy-to-noise ratio is $-1.5917\ dB$.

[2]Recall that $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a, a > 0$.