BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2015 – 16
MATH F 312 : Ordinary Differential Equations
Mid Semester Test (Closed Book)

Duration : 90 mins **Date : 05/10/2015** **MM : 70**

Q1. Does the function $g(t,u) = 3u^{\frac{2}{3}}$ satisfy Lipschitz condition on any domain that intersects the t – axis? Justify your answer. [8]

Q2. Using the method of Picard's iteration, find the first three approximations to the solution of the following IVP:

$$u' = 4t + 2tu, \quad u(0) = 1.$$ [10]

Q3. If $f(t)$ is a continuous function on $0 \le t < \infty$, and $|f(t)| \to 0 \quad as \ t \to \infty$, then show that all solutions of the differential equation

$$u' + \alpha u = f(t), \qquad \alpha > 0,$$

approach zero as $t \to \infty$. [10]

Q4. Find a fundamental matrix $\Phi(t)$ with $\Phi(0) = I$ of the following system of differential equations:

$$x_1' = -3x_1 + x_2,$$
$$x_2' = 2x_1 - 4x_2.$$ [10]

Q3. Let $u(t)$ and $v(t)$ be non-negative continuous functions on some interval $I = [t_0, t_0 + a]$, where a is any fixed positive real number. Also, let the function $f(t)$ be positive, continuous, and monotonically nondecreasing on I and satisfy the inequality

$$u(t) \le f(t) + \int_{t_0}^{t} u(s)v(s)ds, \quad t \in I.$$

Then show that

$$u(t) \le f(t)\exp[\int_{t_0}^{t} v(s)ds], \quad t \in I.$$

[12]

Q6. State and prove Abel-Liouville Formula. [12]

Q7. Determine whether the zero solution of the differential equation

$$u''' + 2u'' + 3u' + 4u = 0$$

is stable or asymptotically stable or unstable? [8]

END