BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2014-15
ME G512: *Finite element method*

Max. time: **180 min.** End-Semester Examination (11/12/2015) Marks: **26**

Instructions

1. **Appropriate/inappropriate assumptions will be appreciated/depriciated. Clearly state them.**

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1. [6 **marks**]

   Consider $2N$th order tensor

   $$\mathcal{C}_{i_1 j_1 i_2 j_2 \cdots i_{N-1} j_{N-1} i_N j_N}$$

   for some positive integer N. It is symmetric in indices i_k and j_k for each $k = 1 \cdots N$, i.e., N minor symmetries.
   It also has a major symmetry, i.e., for even N, we have

   $$\mathcal{C}_{i_1 j_1 \cdots i_{\frac{N}{2}} j_{\frac{N}{2}} i_{\frac{N}{2}+1} j_{\frac{N}{2}+1} \cdots i_N j_N} = \mathcal{C}_{i_{\frac{N}{2}+1} j_{\frac{N}{2}+1} \cdots i_N j_N \cdots i_1 j_1 \cdots i_{\frac{N}{2}} j_{\frac{N}{2}}}.$$

   And for odd N, we have

   $$\mathcal{C}_{i_1 j_1 \cdots i_{\frac{N-1}{2}} j_{\frac{N-1}{2}} i_{\frac{N+1}{2}} j_{\frac{N+1}{2}} \cdots i_N j_N} = \mathcal{C}_{i_{\frac{N+1}{2}} j_{\frac{N+1}{2}} \cdots i_N j_N \cdots i_1 j_1 \cdots i_{\frac{N-1}{2}} j_{\frac{N-1}{2}}}.$$

   What is the number of distinct components of this tensor with these symmetries?

2. [4 **marks**]

   Consider a governing equation

   $$\rho v_j \, v_{i,j} - \sigma_{ij,j} = \rho f_i, \quad \text{over} \quad \Omega,$$

   with

   $$\sigma_{ij} = -p\delta_{ij} + \mu(v_{i,j} + v_{j,i}).$$

   Boundary condition is

   $$v_i = Q \quad \text{over} \quad \Gamma = \partial\Omega.$$

   Find corresponding weak formulation.

3. [$1 + 1.5 + 1.5 = 4$ **marks**]
   Consider

   $$y''' + x^3 = 0, \quad \text{over} \quad (0, 1)$$

   with boundary conditions

   $$y(0) = 0 = y(1) \quad \text{and} \quad y'|_{x=0} = 0.$$

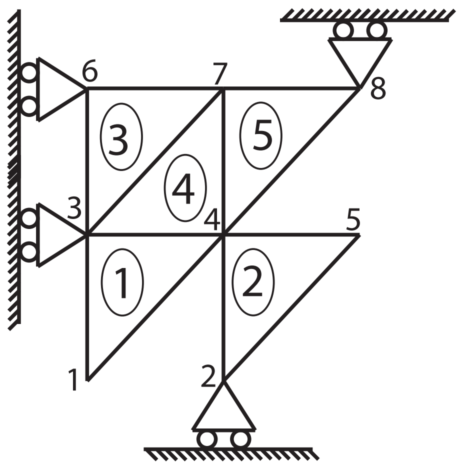

Figure 1: A 2D elastic structure

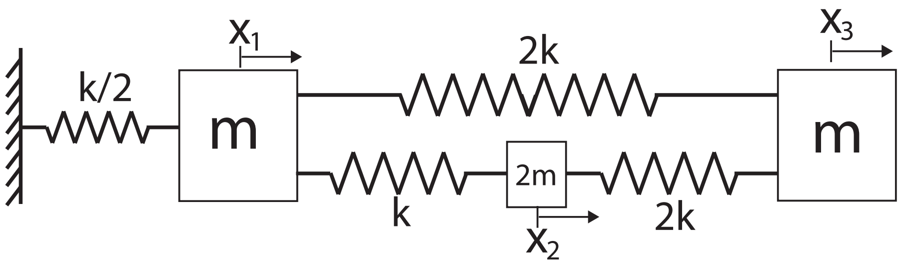

Figure 2: A spring-mass system

(a) Find exact solution to the BVP.

(b) Find approximate solution with at least three shape functions using Bubnov-Galerkin method.

(c) Repeat the above using least squares method.

4. [4 **marks**]

   Refer to an elastic structure as shown in Fig. 1. Derive expression for each diagonal element of the global stiffness matrix in terms of elements of the element stiffness matrices.

5. [$1 + 1.5 + 1.5 = 4$ **marks**]

   Consider a spring-mass system as shown in Fig. 2

   (a) Find mass and stiffness matrices for the system.

   (b) Find Guyan reduced mass and stiffness matrices assuming x_1 and x_2 to be master nodes and x_3 to be a slave node.

   (c) Repeat the above assuming x_2 and x_3 to be master nodes and x_1 to be a slave node.

6. [4 **marks**]

For a six-noded 2D triangular element, shape functions in triangular co-ordinates are given by

$$N_1 = \xi(2\xi - 1), \quad N_2 = \eta(2\eta - 1), \quad N_3 = \zeta(2\zeta - 1)$$

and

$$N_4 = 4\xi\eta, \quad N_2 = 4\eta\zeta, \quad N_3 = 4\zeta\xi, \quad \text{with} \quad \zeta = 1 - \xi - \eta.$$

Using symmetric Gauss quadrature with 3 Gauss points, evaluate

$$I = \int_K xy \, dxdy,$$

K being a triangle with co-ordinates $(1, 1)$, $(-2, -1)$, $(3, 1)$.