BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2015-16
ME G512: *Finite element method*

Time: **1.5 hrs** Mid-Semester Examination (09/10/2015) Marks: **30**

Instructions

1. **Appropriate/inappropriate assumptions will be appreciated/depriciated. Clearly state them.**

1. [$3 + 2 + 1 + 2 = 8$ **marks**]

   From the theory of elastoplastostatics, C_{ijklmn} is a sixth order tensor. When the physical space is n_{sd}-dimensional, indices i, j, k, l, m and n vary over the range 1 to n_{sd}. Minor symmetries of this tensor are

   $$C_{ijklmn} = C_{jiklmn} = C_{ijlkmn} = C_{ijklnm},$$

   and major symmetry is

   $$C_{ijklmn} = C_{lmnijk}.$$

   (a) As we consider four symmetries of this tensor *one after another* for $n_{sd} = 2$, derive successive reduction in number of distinct elements starting from 2^6.

   (b) Considering all symmetries, enlist distinct elements for $n_{sd} = 2$.

   (c) Repeat the above for $n_{sd} = 3$.

   (d) Find an expression for the number of distinct elements in a general case $n_{sd} = N$.

2. [6 **marks**]

   Refer to Fig. 1. Derive expression for each diagonal element of the global stiffness matrix in terms of elements of ten element stiffness matrices.
   Note that portion of the boundary joining nodes with global node numbering 3-4-5, 8-11, 14-15 and 17-18-19 is of Dirichlet type and the rest is of Neumann type.

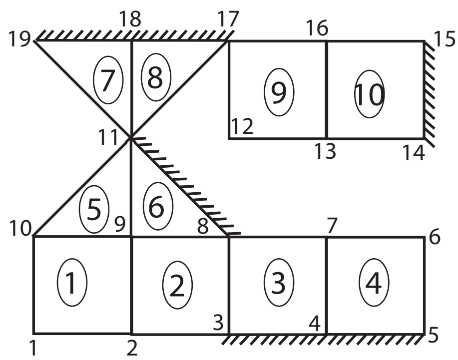

Figure 1: Discretized domain for the problem of linear heat conduction

3. [$1+4+1=6$ **marks**]

Consider a boundary value problem with governing equation

$$\alpha_{ijk,k} = \mu\beta_{ij}$$

and a constitative equation

$$\alpha_{ijk} = \tau_{ijklmn}\,\gamma_{lmn},$$

with

$$\gamma_{lmn} = \frac{1}{6}\big(\phi_{lm,n} + \phi_{nl,m} + \phi_{mn,l} + \phi_{ml,n} + \phi_{ln,m} + \phi_{nm,l}\big),$$

and boundary conditions

$$\alpha_{ijk}n_k = \Delta_{ij} \quad \text{over} \quad \Gamma_{\Delta_{ij}}$$

and

$$\phi_{ij} = \Phi_{ij} \quad \text{over} \quad \Gamma_{\Phi_{ij}}$$

with usual understanding of

$$\Gamma_{\Phi_{ij}} \cup \Gamma_{\Delta_{ij}} = \Gamma = \partial\Omega \quad \text{and} \quad \Gamma_{\Phi_{ij}} \cap \Gamma_{\Delta_{ij}} = \emptyset \quad \text{for every } \; i, j.$$

(a) Define spaces of trial and weighting functions.

(b) Derive corresponding weak form.

(c) Write neatly the weak statement.

4. [$1+3+1+1+1+2+1=10$ **marks**]

Consider a boundary value problem with governing equation

$$u_{j,ii} = f_j, \qquad j = 1 \;\text{ to }\; n_{sd},\; n_{sd} > 1,$$

with a boundary condition

$$-u_{j,i}n_i = \bar{h}_j \;\text{ over }\; \Gamma.$$

(a) Define spaces of trial and weighting functions.

(b) Derive corresponding weak form.

(c) Write neatly the weak statement.

(d) Derive Galerkin form.

(e) Write neatly the Galerkin statement.

(f) Derive an arbitrary entry of the global stiffness matrix (k_{mn}).

(g) Derive the corresponding entry of the force vector (f_m).