Birla Institute of Technology & Science
Pilani, Rajasthan- 333 031
(First semester 2015 - 2016)

ME G535 Advanced Engineering Mathematics
End-sem Exam (open notes)

1st December, 2015

Max. marks - 40 Duration:- min 180 min.

1. [2+3+1=6 marks]

   A particle of mass m which is constrained to move in the x-y plane (refer to Fig. 1), and is restrained by two linear springs, each with spring constant of k. The anchor points of the two springs are located on the x axis at $x = 1$ and $x = 1$. Each of the two springs has unstretched length L.

   (a) Derive the Lagrangian fo the system.

   (b) Using the Lagrangian, derive equations governing motion of the particle.

   (c) Describe mathematically equations so obtained.

2. [6 marks]

   Prove the following using indicial notation.

$$\nabla^2\Big(\nabla\big(\nabla\cdot(\omega\times r)\big)\Big) = \nabla\big(\nabla\cdot\nabla^2(\omega\times r)\big) = \nabla\Big(\nabla\cdot\Big(\nabla\big(\nabla\cdot(\omega\times r)\big)\Big)\Big)$$

3. [1+2+2=5 marks]

   Consider Bessel's equation

$$y'' + \frac{1}{x}y' + \Big(1 - \frac{\nu^2}{x^2}\Big)y = 0.$$

   (a) Is $x = 0$ a singular point or not? If so, is it regular singular or not?

   (b) Choose the method accordingly to find at least one independent solution to the equation for $\nu = 1/2$.

   (c) Repeat the above for $\nu = 0$.

4. [1+3 = 4 marks]

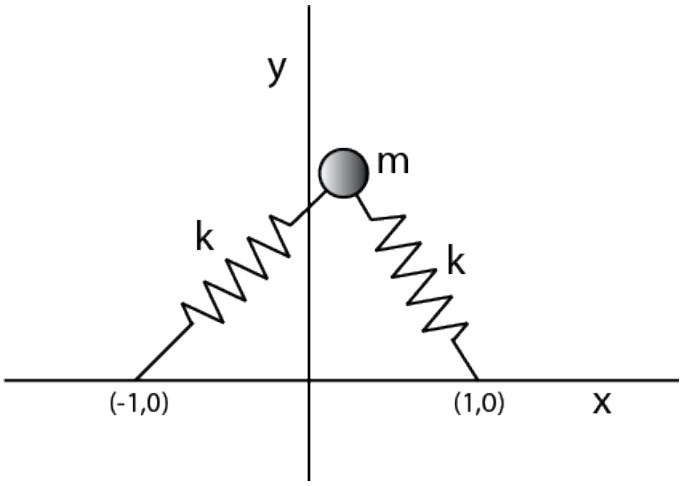

Figure 1: A particle in a plane

(a) Plot a graph of $x^{1/x}$ over $(0, \infty)$.

(b) Using the graph, get at least one solution to $x^y = y^x$.

5. [1+1+1 = 3 marks]

A vectorial invariant for a SOT T_{ij} is defined as

$$T_\times = T_{ij}\, e_i \times e_j.$$

Find the vectorial invariant of the following SOTs

(a) δ_{ij},

(b) T_{ij} if $T_{ij} = T_{ji}$,

(c) $a_i b_j - a_j b_i$.

6. [4 marks]

Prove Euler's theorem of 3D rotations: The most general displacement of a rigid body about one point fixed is rotation about some axis passing through that point.

7. [4 marks]

Starting with

$$\epsilon_{ij,kl} + \epsilon_{kl,ij} - \epsilon_{ik,jl} - \epsilon_{jl,ik},$$

derive Mitchell-Beltrami compatibility equations

$$T_{ij,kk} + \frac{1}{1+\nu} T_{pp,ij} = -\frac{\nu}{1-\nu}\delta_{ij}(\rho b_j),_j - (\rho b_i),_j - (\rho b_j),_i \,.$$

8. [3 marks]

   If

$$f(x) + f(\frac{x-1}{x}) = 1 + x,$$

   find $f(x)$.

9. [3 marks]

   Using Gram-Schmidt procedure, construct an orthonormal basis for vectors

$$w_1 = \{1\ 1\ -1\}^T, \quad w_2 = \{1\ 0\ 2\}^T, \quad w_3 = \{2\ -2\ 3\}^T.$$

10. [2 marks]

    Plot actual error and estimated truncation error for a centered difference approximation to $\frac{d}{dx}\sin(x)$ at $x = 1$.