

Birla Institute of Technology & Science  
Pilani, Rajasthan- 333 031  
(First semester 2015 - 2016)

**ME G535 Advanced Engineering Mathematics**  
**Mid-sem Exam (open notes)**

**November 26, 2015**

Max. marks - 30      Duration:- 90 min.

1. [5 marks]

Quaternion representation of a 3D rotation about  $\mathbf{n} = \{n_1, n_2, n_3\}^T$  through  $\theta$  is

$$\mathbf{q} = \cos(\theta/2) + \sin(\theta/2) \left( n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k} \right).$$

Important identities of quaternion algebra are

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ij} = \mathbf{k}, \quad \mathbf{jk} = \mathbf{i}, \quad \mathbf{ki} = \mathbf{j}, \quad \mathbf{ji} = -\mathbf{k}, \quad \mathbf{kj} = -\mathbf{i}, \quad \mathbf{ik} = -\mathbf{j}.$$

Rotated vector  $\tilde{\mathbf{v}}$  is given by  $\tilde{\mathbf{v}} = \mathbf{q}\mathbf{v}\mathbf{q}^{-1}$ . Find out  $\mathbf{q}^{-1}$  using above identities and axis-angle formula.

2. [2+3 = 5 marks]

- (a) What is the trace of a  $2 \times 2$  symmetric orthogonal matrix with determinant 1?
- (b) What is the trace, other conditions being same, if the matrix is  $3 \times 3$ ?

3. [5 marks]

Consider

$$\dot{\mathbf{x}} = A\mathbf{x},$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}.$$

What is the fundamental solution matrix?

4. [5 marks]

Eigenvalues of a rotation matrix are 1,  $i$ ,  $-i$ . What is the angle of rotation and axis of rotation? Write down the rotation matrix.

5. [5 marks]

Is

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

diagonalizable? If yes, find the corresponding diagonal matrix. If no, find the corresponding Jordan canonical form.

6. [2.5 + 2.5 = 5 marks]

What are the algebraic and geometric multiplicities of all eigenvalues of a  $3 \times 3$  shear matrix that fixes  $x = y$  plane?