

Date : 12 Dec 2015

Max. Marks : 40

**A**

Duration : 30 mins

Name :

ID. :

Each question carry 2 marks.

- For steady currents, which one of the statement is true ?  
(A)  $\mathbf{J} = 0$  (B)  $\nabla \cdot \mathbf{J} = 0$  (C)  $\nabla \times \mathbf{J} = 0$  (D) none of these
- The normal component of the electric field at  $r = R$  of a metallic spherical shell of radius  $R$  which is centered at the origin and carries a charge  $Q$  is discontinuous by an amount:  
(A)  $Q/(4\pi\epsilon_0 R^2)$  (B)  $Q/(8\pi\epsilon_0 R^2)$  (C)  $Q/(2\pi\epsilon_0 R^2)$  (D)  $Q/(\pi\epsilon_0 R^2)$
- A square loop of wire with length and width  $d$  centred at the origin sits in the  $x-y$  plane and has a current  $I$  passing through it. What is its magnetic dipole moment?  
(A)  $\mathbf{m} = Id^2 \hat{\mathbf{x}}$  (B)  $\mathbf{m} = 2\pi Id \hat{\mathbf{z}}$  (C)  $\mathbf{m} = Id^2 \hat{\mathbf{z}}$  (D)  $\mathbf{m} = I\pi d^2 \hat{\mathbf{z}}$
- In some material the magnetization is defined as  $\mathbf{M} = 2y \hat{\mathbf{x}} + 3z \hat{\mathbf{y}} + 5xyz \hat{\mathbf{z}}$ . The magnitude of volume current density at  $(-1, 2, 1)$ .  
(A)  $\sqrt{108}$  (B)  $2\sqrt{42}$  (C)  $\sqrt{253}$  (D)  $\sqrt{273}$
- The tangential component of the electric field on the either side of the metallic sheet which carries a surface charge density  $\sigma$  is discontinuous by an amount:  
(A)  $\sigma/(2\epsilon_0)$  (B)  $\sigma/(4\epsilon_0)$  (C)  $\sigma/(6\epsilon_0)$  (D) 0
- Energy of a uniformly charged sphere of total charge  $q$  and radius  $R$  is:  
(A)  $3q^2/(8\pi\epsilon_0 R)$  (B)  $3q^2/(16\pi\epsilon_0 R)$  (C)  $3q^2/(20\pi\epsilon_0 R)$  (D)  $3q^2/(32\pi\epsilon_0 R)$
- Which one of the boundary condition is correct.  
(A)  $B_{above}^{\parallel} - B_{below}^{\parallel} = -(M_{above}^{\parallel} - M_{below}^{\parallel})$   
(B)  $H_{above}^{\parallel} - H_{below}^{\parallel} = -(M_{above}^{\parallel} - M_{below}^{\parallel})$   
(C)  $B_{above}^{\perp} - B_{below}^{\perp} = -(M_{above}^{\perp} - M_{below}^{\perp})$   
(D)  $H_{above}^{\perp} - H_{below}^{\perp} = -(M_{above}^{\perp} - M_{below}^{\perp})$
- A long straight wire carries a uniform line charge  $\lambda$ , is surrounded by rubber insulation out to a radius  $a$ . The expression for the electric displacement is given as:  
(A)  $\vec{D} = \lambda/(2\pi s) \hat{s}$  (B)  $\vec{D} = \lambda/(2\pi\epsilon_0 s) \hat{s}$  (C)  $\vec{D} = 2\pi\lambda/(\epsilon_0 s) \hat{s}$  (D)  $\vec{D} = 4\pi\lambda/(\epsilon_0 s) \hat{s}$
- Two infinite parallel plates carry equal but opposite uniform charge densities  $\pm\sigma$ . The field between the plates is:  
(A)  $\sigma/(2\epsilon_0)$  (B)  $\sigma/\epsilon_0$  (C)  $2\sigma/\epsilon_0$  (D)  $\sigma/(4\epsilon_0)$
- The capacitance of two concentric spherical shells with radii  $a$  and  $b$  is:  
(A)  $4\pi\epsilon_0 ab/(a-b)$  (B)  $ab/[4\pi\epsilon_0(a-b)]$  (C)  $4\pi\epsilon_0 (a-b)/ab$  (D)  $(a-b)/[4\pi\epsilon_0 ab]$
- The value of the integral,  
$$I = \int_{-1}^1 9x^2 \delta(3x+1) dx$$
  
(A)  $-1/3$  (B)  $1/9$  (C)  $1/27$  (D)  $1/3$

12. The potential of a uniformly charged spherical shell of total charge  $q$  and of radius  $R$  at a distance  $r$  ( $r < R$ ) from its origin is:  
 (A)  $V = q/(4\pi\epsilon_0 R)$  (B)  $V = q/(4\pi\epsilon_0 r)$  (C)  $V = qR/(4\pi\epsilon_0 r^2)$  (D)  $V = qr/(4\pi\epsilon_0 R^2)$
13. The force of attraction between a grounded conducting sphere of radius  $R$  and a point charge  $q$  which is situated at a distance  $b$  ( $b > R$ ) from the center of the above grounded sphere is:  
 (A)  $F = -\frac{1}{4\pi\epsilon_0} \frac{q^2 R b}{(b^2 - R^2)^2}$  (B)  $F = -\frac{1}{4\pi\epsilon_0} \frac{q^2 R^2 b^2}{(b^2 - R^2)^4}$  (C)  $F = -\frac{1}{4\pi\epsilon_0} \frac{q^2 R b}{(b^2 - R^2)}$  (D)  $F = -\frac{1}{4\pi\epsilon_0} \frac{q^2 R^4 b^2}{(b^2 - R^2)^2}$
14. Which one of the statement is not correct.  
 (A) The permanent magnets are classified as ferromagnetic materials.  
 (B) In diamagnetic materials dipoles are aligned along the direction opposite to applied magnetic field.  
 (C) In paramagnetic materials dipoles are aligned along the direction of applied magnetic field.  
 (D) Ferromagnetism involves the magnetic dipoles associated with the orbital motion of the electron.
15. If a uniform magnetic field  $\mathbf{B} = B_0 \hat{z}$  is applied to the loop in Question (3). What would be the net force experienced by the loop.  
 (A)  $F = B_0 I d$  (B)  $F = 0$  (C)  $F = 4B_0 I d$  (D)  $F = B_0 I z^2 / d$
16. Energy of a uniformly charged spherical shell of total charge  $q$  and radius  $R$  is:  
 (A)  $q^2/(8\pi\epsilon_0 R)$  (B)  $q^2/(16\pi\epsilon_0 R)$  (C)  $q^2/(24\pi\epsilon_0 R)$  (D)  $q^2/(32\pi\epsilon_0 R)$
17. The magnetic field as given in Question (18) can be obtained by following vector potential,  
 (A)  $\mathbf{A} = 0.05 z \hat{x} - 0.125 z \hat{y} + 0.125 y \hat{z}$   
 (B)  $\mathbf{A} = 0.05 y \hat{x} + 0.125 z \hat{y} + 0.125 x \hat{z}$   
 (C)  $\mathbf{A} = 0.05 z \hat{x} + 0.125 y \hat{y} - 0.125 x \hat{z}$   
 (D)  $\mathbf{A} = 0.05 x \hat{x} - 0.125 x \hat{y} - 0.125 z \hat{z}$
18. A test particle having unit mass and unit positive charge is moving in free space such that its velocity vector is defined by  $\mathbf{v} = 2\hat{x} + 5\hat{y} - 3\hat{z}$  (SI units). If this particle is subjected to a constant magnetic field  $\mathbf{B} = 0.25 \hat{x} + 0.05 \hat{y}$  (SI units) in a cubical (each side 2 cm) region around the origin. What will be the energy of the particle when it leaves the said region.  
 (A) 0.25 Joules (B) 15 Joules (C) 19 Joules (D) 0 Joules
19. Among following which expression is not correct in the context of magnetostatics.  
 (A)  $\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$  (B)  $\nabla \times \mathbf{H} = \mathbf{J}_f$  (C)  $\mathbf{K}_b = \hat{n} \times \mathbf{M}$  (D)  $\nabla \cdot \mathbf{J}_b = 0$
20. If magnetic monopoles did exist, how many of Maxwell's equations would have to be changed ?  
 (A) one (B) two (C) none (D) all of them

Date : 12 Dec 2015

Question:	1	2	3	4	5	6	7	8	9	10
Answer:										
Question:	11	12	13	14	15	16	17	18	19	20
Answer:										

**A**

For evaluation purpose only. Do not write here.

Correct Answers:

Marks Obtained:

Date : 12 Dec 2015  
Max. Marks : 80

Duration : 2.5 hours

- Solve **Part-A** and **Part-B** on separate answer sheets.
- You should strictly follow the geometry and the coordinate system as given in the problem. The argument regarding the use of different coordinate system will not be entertained during recheck.
- Use proper vector signs otherwise marks will be deducted.

**Part - A**

1. The charge density of a certain configuration centered at the origin and which is also spherically symmetric is given as,

$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a}$$

where  $a$  is a constant.

- (a) Determine the expression for the potential  $V(r)$  at a distance  $r$  from the origin due to the electric field produced by this charge configuration.
- (b) From the expression of  $V(r)$ , calculate the expression of the electric field,  $E(r)$ .

[10+5]

2. A sphere of radius  $R$  is uniformly charged with a volume charge density  $\rho$ . Inside the sphere, there is a spherical cavity of radius  $b$  whose center is at a distance  $a$  from the center of the sphere as shown in the Figure 1 below. Calculate the expression of the electric field at a point,

- (a) Inside the cavity,  
(b) outside the cavity but inside the sphere and  
(c) outside the sphere.

[5+5+5]

3. A metal sphere of radius  $a$  carries a free charge  $Q$  (Figure 2). It is surrounded out to radius  $b$  and  $c$  by concentric layers of linear dielectric material of permittivity  $\epsilon_1$  and  $\epsilon_2$  respectively. Calculate the expression for the potential at the center of this configuration relative to infinity.

[10]

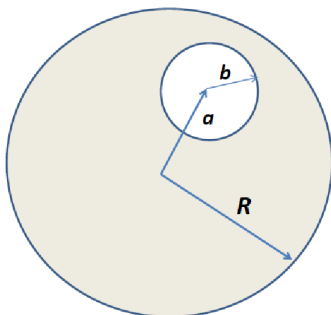


Figure 1

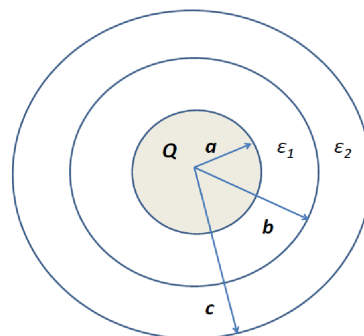


Figure 2

**Part - B**

1. A test particle of mass  $m$  and charge  $q$  is placed at  $x = y = z = 0$ . The particle is initially at rest. A constant magnetic field  $B_0$  is applied along the  $x$  direction, however a constant electric field of magnitude  $E_0$  is applied at an angle of  $45^\circ$  with applied magnetic field in  $xy$  plane.
- (a) Obtain an expression for the time dependent energy of the particle.
  - (b) Calculate the trajectory of the particle i.e.  $x(t), y(t)$  and  $z(t)$ .
  - (c) What will be the radius of the cycloid as seen from the  $yz$  plane.

**[10+10+5]**

2. (a) A square loop of wire, with sides of length of  $a$ , lies in the first quadrant of the  $xy$  plane, with one corner at the origin. In this region there is a nonuniform time-dependent magnetic field  $\mathbf{B}(y,t) = ky^3t^2\hat{\mathbf{z}}$  (where  $k$  is constant). Find the emf induced in the loop.
- (b) An infinitely long cylinder, of radius  $R$ , carries a magnetization, parallel to axis,  $\mathbf{M} = ks\hat{\mathbf{z}}$ , where  $k$  is a constant and  $s$  is the distance from the axis; there is no free current anywhere, find the magnetic field  $\mathbf{B}$ , inside and outside the cylinder.
- (c) Write down with proper explanation the four set of Maxwell's equations. What was the problem with the Ampere's law of magnetostatics and how it was fixed by Maxwell ?

**[6+5+4]**

---