Birla Institute of Technology & Science, Pilani

First Semester 2017-2018, BITS F113 (General Mathematics I)
Comprehensive Examination

Time: 3 Hours **Date: December 09, 2017 (Saturday)** **Max. Marks: 45**

1. The question paper consists of two parts. **Part A (Closed Book)** is of 25 Marks & 100 Minutes and **Part B (Open Book)** is of 20 Marks & 80 Minutes. Attempt questions of **Part A** and **Part B** on two separate answer sheets.
2. Answer sheet and question paper of **Part B** shall be given only after the submission of **Part A** answer sheet. Late submission of **Part A** is not allowed. On the top right corner of the first answer sheet, write **Part A**, and on the second answer sheet, write **Part B**.
3. Begin solution of each question on a new page, and answer the parts (if any) of each question in continuation. **Calculator is not allowed**. Write **END** at the end of the last attempted solution in each answer sheet. Use ball point pen only for writing the solutions and drawing the diagrams.

Part A (Closed Book) **Time: 2:00PM-3:40PM (100 Min.)** **Max. Marks: 25**

Q. 1 (a) Draw appropriate Venn diagrams for $(A \cap B)'$, $(A \cup B)'$, $A' \cup B'$ and $A' \cap B'$.

(b) Describe the Modulus, Signum and Greatest Integer functions along with the domain, range and graph of each function. **[2+3]**

Q. 2 (a) Express $\cos 6x$ in terms of $\cos x$.

(b) Convert the complex number $\frac{-16}{1+i\sqrt{3}}$ into polar form.

(c) Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$. **[2+2+1]**

Q. 3 (a) In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

(b) The second, third and fourth terms in the binomial expansion $(x + a)^n$ are 240, 720 and 1080, respectively. Find a and n.

(c) The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the first term and common ratio. **[2+2+1]**

Q. 4 (a) Solve (i) $\lim_{x\to 1}\left(\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x}\right)$ (ii) $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$.

(b) Find $\frac{dy}{dx}$ from $\tan^{-1}\left(\frac{y}{x}\right) + 2^y + xy = 5$.

(c) Find $\frac{dy}{dx}$ from $y = x^{\ln x}$. **[2+2+1]**

Q. 5 (a) Find the equations of the tangent and normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$.

(b) A box without top is to be made from a square piece of tin of side 18 cm by cutting a square from its each corner and folding up the flaps. What should be the side of the square to be cut off so that volume of the box is maximum? **[2+3]**

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Note: For the open book examination, the text books (NCERT Class XI and XII) and hand written class notes are allowed. Any other printed or photocopy material is not allowed. Loose sheets of hand written notes are also not allowed.

Part B (Open Book) **Time: 3:40PM-5:00PM (80 Min.)** **Max. Marks: 20**

Q. 1 Solve the following integrals:

(i) $\int \tan^{-1}\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right) dx$

(ii) $\int \text{cosec}^6 x \, dx.$ **[3+2]**

Q. 2 Solve $\int \frac{e^x}{e^{3x}+1} dx.$ **[5]**

Q. 3 Evaluate $\int_0^{\pi/2} \frac{x}{1+\sin x+\cos x} dx.$ **[5]**

Q. 4 Sketch and find the area of that part of the circle $x^2+y^2=16$ which is exterior to the parabola $y^2=6x$. **[5]**