Birla Institute of Technology & Science, Pilani
First Semester 2022 – 2023
Comprehensive Examination
General Mathematics I (BITS F113)

Date: February 21, 2023 **Max. Time: 180 Minutes** **Max. Marks: 40**

Max. Marks: 20 **Part- A Closed Book** **Max. Time: 90 minutes**

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1. (a) In a group of 70 people, 37 like coffee, 52 like tea and each person like atleast one of the two drinks. How many people like coffee but not tea? **[2]**

   (b) Find the range of the function $f(x) = x^2 + 2$, x is a real number. **[1]**

   (c) Determine the domain of the function $f(x) = \dfrac{1}{x} + \sin^{-1} x$. **[1]**

2. Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 4i$. **[3]**

3. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$. **[2]**

4. Determine the equation of the hyperbola whose vertices are $(\pm 2, 0)$ and foci are $(\pm 3, 0)$. **[2]**

5. (a) Find the value of $\dfrac{dy}{dx} + 2023$ if $y = \cos^{-1}(\sin x)$. **[3]**

   (b) State Rolle's theorem and verify it for the function

   $$f(x) = x^4 + x^2 + 2, \ x \in [-2, 2].$$ **[3]**

   (c) Using differentials, find the approximate value of $\left(\dfrac{17}{81}\right)^{\frac{1}{4}}$. **[3]**

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1. Find the general solution of $\cos x + \cos 3x + \cos 5x = 0$. **[3]**

2. In how many ways 5 girls and 4 boys be seated in a row so that no two girls are together? **[2]**

3. Find the coefficient of x^5 in the product $(1 + x)^3(1 + x^2)^4$. **[2]**

4. If a, b, c, d are in GP, prove/disprove that $d^n - c^n, c^n - b^n, b^n - a^n$ are in GP. **[2]**

5. Find the coordinates of the foot of perpendicular from the point $(3, -1)$ to the line $4x - 3y = 16$. **[3]**

6. (a) Evaluate $\int_1^4 \left[\sin(\pi x) + |x - 2| + \frac{x^2 + x + 1}{(x + 1)^2(x + 2)}\right] dx$. **[5]**

   (b) Find the area of the region $\{(x, y) : 0 \leq x \leq 4y, 4x^2 + 4y^2 \leq 9\}$. **[3]**

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