BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Mid-semester Exam, 2023-24

General Mathematics I (BITS F113)

Oct 11, 2023

Max. Time : 90 mins **Max. Marks: 30**

Name :..... **ID No. :**.....

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1. If $\sin(A) = 0.6$ and $\cos(B) = 0.3$, find $\sin(A + B)$ using trigonometric identities. [2]

2. Compute the product of the complex numbers: $(-2 + 3i)$ and $(4 - i)$. Find the complex conjugate of the product. [1+1]

3. Determine the square root of the complex number $z = 4 + 3i$, and discuss whether the resulting number is less than, greater than, or equal to z. [1+1]

4. In how many ways can you arrange the letters in the word "COMBINATIONS"? [2]

5. Prove or disprove the identity: $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ for $-1 \leq x \leq 1$. [2]

6. The arithmetic mean of two positive numbers is 8, and the geometric mean is 4. Evaluate the two numbers. [2]

7. Find the expansion of $(x - 2)^5$ up to the 4th term, using the binomial series expansion. [3]

8. Suppose you have a table of temperature values at various altitudes. The table shows temperatures at altitudes in 1000-foot increments. The temperature at 0 feet (sea level) is $70°$F, and at 1000 feet it's $65°$F. Using linear interpolation, estimate the temperature at an altitude of 750 feet. [3]

9. Consider the equation of a parabola: $y = ax^2 + bx + c$, where a, b, and c are real constants. [2+2+2]

   (a) If the vertex of this parabola is at the point $(2, -3)$, find the values of a, b, and c.

   (b) Determine the axis of symmetry for this parabola.

   (c) Find the coordinates of the focus and the equation of the directrix for this parabola.

10. Consider the equation of an ellipse in standard form: [3+3]

$$\frac{(x-3)^2}{9} + \frac{(y+2)^2}{4} = 1$$

   (a) Determine the center, semi-major axis length, and semi-minor axis length of this ellipse.

   (b) Sketch the ellipse on a coordinate plane, labeling the major and minor axes, and indicating the center.