Birla Institute of Technology & Science, Pilani (Raj.) Second Semester 2016-2017, BITS F114 (General Mathematics II) End Semester Examination

Time: 180 Min. Date: May 05, 2017 (Friday) Max. Marks: 40

- 1. Write solution of each question on fresh page.
- 2. All questions are compulsory and carry equal marks.

CLOSED BOOK QUESTIONS

- Q. 1 Show that the function $v(x, y) = e^x \sin y$ is harmonic. Find its conjugate harmonic function u(x, y).
- Q. 2 Show that the function

$$f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & z \neq 0\\ 0 & z = 0 \end{cases}$$

is not differentiable at z = 0.

- Q. 3 Solve $\sec^2 y \frac{dy}{dx} + \tan y = x^3$.
- Q. 4 Find integrating factor so that the differential equation $(3x^2 y^2)dy 2xydx = 0$ is exact, and hence solve it.
- Q. 5 Use the method of variation of parameters to solve $y'' + y = \csc x$.

OPEN BOOK QUESTIONS

Q. 1 Find $f_x(0, y)$ and $f_y(x, 0)$ from

$$f(x,y) = \begin{cases} \frac{xy(2x^2 - 3y^2)}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

- Q. 2 Find the extremum values of $x^3 + y^3 3axy$.
- Q. 3 Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$, and hence evaluate the same.
- Q. 4 Use Laplace transform to solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos 2t \text{ with the conditions } x(0) = 1 \text{ and } x(\pi/2) = -1.$
- Q. 5 Find the inverse Laplace transform of $\frac{s-1}{s(s^2+1)(s^2+2)}$.