<u>**Date: 04 April, 2023**</u>　　　　　　　　**Time: 90 Minutes**　　　　　　　　<u>**Max. Marks: 25**</u>

Note:
• Notations/symbols have their usual meaning.
• Start new question on a fresh page. Moreover, answer each subpart of a question in continuation.
• Draw the figures as and when required.

Q.1(A). Discuss the *symmetries* of the *polar curve* $r = 1 + \sin\theta$. Find all the points of *intersections* of this curve with the curve $r = 3(1 - \sin\theta)$.

Q.1(B). Sketch the *polar curve* $r = \frac{12}{3+\sin\theta}$. Find the *directrix* and *label* the *center*, *focus* and all the *vertices* in the *polar coordinate*.　　　　　　　　　　　　　　　　　[4+4]

Q.2. The position of a *moving particle* in the *xy*-plane at time t is $\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}$. Find the *time* when the particle achieves the *maximum speed*. Also find *maximum speed*.　　　[4]

Q.3. Consider the following function:
$$f(x,y) = \frac{xy + y^3}{x^2 + y^2}$$

(A) Find the set *A* of points where *f* is *discontinuous*.

(B) Can we *redefine f* on $\mathbb{R}^2$ in a manner so that it would become *continuous* on *A*? Justify.

(C) Assume $f(0,0) = 0$, does the *directional derivative* at *origin* in the *direction* of $\vec{u} = 0\hat{i} + 5\hat{j}$ exist? Justify?　　　　　　　　　　　　　　　　　　　　　　　　　　[2+3+4]

Q.4. For what values of the constant k, $f(x,y) = x^2 + +kxy + y^2$ has a *local minimum* at (0,0)? [4]