BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Second Semester 2022-23
BITS F114 (General Mathematics - II)
Comprehensive Examination-Part A (Closed Book)

Date: 10th July, 2023 **Time: 2 Hrs** **Max. Marks: 30**

Note:

- Notations/symbols have their usual meaning.
- Start new question on a fresh page. Moreover, answer each subpart of a question in continuation.
- Draw the figures as and when required.

Q.1. Prove or disprove that if the *speed* of a particle *moving in a plane* is *constant*, then its *acceleration* is *zero*. [3]

Q.2. Draw the curve $r = (1/2) + \cos\theta,\ 0 \le \theta \le 2\pi$. [3]

Q.3. If $\frac{1}{w} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$, then determine $\frac{\partial w}{\partial y}$ at (30, 45, 90). [3]

Q.4. Sketch the region of integration and evaluate

$$I = \int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy dx.$$ [4]

Q.5. Solve the differential equation $(2x^2 + y)dx + (x^2 y - x)dy = 0$ [4]

Q.6. Transform the differential equation $(y+1)\frac{dy}{dx} + x(y^2 + 2y) = x$ in to a linear differential equation of first order and then find the solution. [4]

Q.7. Given that $y = x + 1$ is a solution of $(x+1)^2 \frac{d^2 y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0$. Find the other *linearly independent* solution. [4]

Q.8. Find the general solution of $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$ by method of *undetermined coefficients*. [5]

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Comprehensive Examination-Part B (Open Book)

Date: 10th July, 2023   Time: 60 Min   Max. Marks: 15

Q.1. If f and g are two analytic functions defined on a common domain $\mathbb{D}$ such that:

Re (f (z)) = Re (g (z)), for all z in $\mathbb{D}$, then show that $f(z) = g(z) + c$, where c is a pure imaginary constant. [5]

Q.2. Let

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ t-\pi, & \pi \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases},$$

write $f(t)$ with the help of *Heaviside unit step* function, and compute *Laplace transform* of $f(t)$ by using *second shift* property. [5]

Q.3. Use *Laplace transform* method to solve the *initial value problem*

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 6e^{-t}, \;\; y(0) = 1, \;\; y'(0) = 2.$$ [5]