

Birla Institute of Technology and Science, Pilani
First Semester 2022-2023
BITS F218: General Mathematics III
Mid-Semester Exam (Closed book)

Max Marks 35

Max time: 90 mins

Date: 02-11-2022

Note:

1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
 2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
 3. Assumptions made if any, should be stated clearly at the beginning of your answer.
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Q.1. Consider the following system of linear equations:

$$\begin{aligned}x_1 + 3x_2 + x_3 &= 6 \\2x_1 + 6x_2 + 3x_3 &= 16 \\3x_1 + 9x_2 + 4x_3 &= 22\end{aligned}$$

- (a) Express above system in the form $AX = b$ and write the augmented matrix.
- (b) Reduce the augmented matrix in the row echelon form (REF).
- (c) Write the rank of the coefficient matrix A.
- (d) Using part (b), write down the solution (s) of the given system, identify the independent variable if any. [8]

Q.2. Is $W = \{(x_1, x_1 + x_2, x_2) \mid x_1, x_2 \in \mathbb{R}\}$ a subspace of \mathbb{R}^3 under usual addition and multiplication operations? [4]

Q.3. Show that the mapping $T : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $T(x_1, x_2) = x_1^2 + x_2^2$ is not a linear transformation. [4]

Q.4. Do $(1, 2, 3), (4, 5, 6), (7, 8, 9)$ form a basis for \mathbb{R}^3 ? Justify your answer. [2]

Q.5. Find the eigenvalues of A where $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$ [3]

Q.6. A company manufactures two products A and B. These products are processed in same machine. It takes 8 minutes to process one unit of product A and 5 minutes for each unit of product B. The machine operates for maximum 34 hours in a week. Product A requires 1 kg. and B requires 0.5 kg. of raw material per unit. The supply of raw material is 600 kg per week. Market demand for product A and B is estimated at least 500 and 600 units respectively every week. Product A costs Rs. 6 per unit and sold at Rs. 10. Product B costs Rs. 8 per unit and can be sold in the market at a unit price of Rs. 11. Formulate the above problem as linear programming Problem to maximize the profit per week. [6]

Q.7. Using Simplex method find all optimal solutions (if exist) of the following LPP

Maximize $z = 2x_1 + x_2 - x_3$
subject to

$$\begin{aligned}-4x_1 + 5x_2 + 2x_3 &\leq 60 \\x_1 + 4x_3 &\leq 36 \\2x_1 - 3x_2 - x_3 &\geq -24 \\x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0\end{aligned}$$

(Show all calculations in tabular form and header of the table should be in textbook format)

[8]

****END****