Birla Institute of Technology & Science, Pilani

First Semester 2022-2023
BITS F218: General Mathematics III
End-Semester Exam (Closed book)

Note :

(i) Question Paper is divided into two parts, **PART A** and **PART B**. **PART A** consist of multiple choice questions and **PART B** is descriptive.

(ii) **PART B** question paper will be given only after submission of **PART A.**

(iii) Rough work should be done at the end of answer book provided and finally it should be crossed.

(iv) Write your name and ID number in the space given in PART A.

(v) There are **TEN** questions in **PART A.** Write the most appropriate answer in the box provided below and nowhere else. Each **correct answer** carries **1 marks** and **wrong answer (−0.25)** mark. **Overwriting/cutting will carry ZERO credit.**

ID No.	Name	Signature od Invigilator

Part A

Max Marks 10 **Max time: 30 mins** **Date: 23-12-2022**

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.										

1. If the homogeneous system $AX = 0$ has a non-trivial solution, then

   (A) $|A| \neq 0$, (B) $|A| > 0$, (C) $|A| = 0$, (D) $|A| < 0$

2. Let A and B be row-equivalent matrices, then

   (A) A is invertible iff B is invertible, (B) $\det(A) = \det(B)$

   (C) $\text{rank}(A) \geq \text{rank}(B)$, (D) $\text{rank}(A) \leq \text{rank}(B)$

3. If eigen values of $A = \begin{bmatrix} 2 & 3 \\ b & a \end{bmatrix}$ are 4 and 8, then

   (A) $a = 10,\ b = 4$, (B) $a = 10,\ b = -4$, (C) $a = 9,\ b = -3$, (D) $a = 8,\ b = 5$.

4. ---------- are expressed in the form of inequities or equations

   (A) Constraints (B) Objective Functions (C) Both A and B (D) None of these.

P.T.O.

5. For the transportation problem

\$2	\$1	\$4	6
\$2	\$1	\$3	7
\$2	\$5	\$2	7
5	5	10	

the initial total cost using North-West corner Method is _____ .

(A) 41 (B) 38 (C) 35 (D) None of these.

6. Every LPP is associated with another LPP is called

(A) Primal (B) Dual (C) Non-linear programming (D) None of these.

7. If the optimal value of the objective function of a LPP with feasible region: $2x_1 + 3x_2 \leq 20$, $x_1, x_2 \geq 0$ is given by 80, then the optimal solution of the dual is

(A) $y_1 = 4$ $y_2 = 2$, (B) $y_2 = 2$, (C) $y_1 = 4$, (D) $y_1 = 2$, $y_2 = 4$.

8. Consider the following LP model: Maximize $z = -x_1 + 3x_2$
s.t. $x_1 + 2x_2 \geq 4; x_1 + 2x_2 \geq 3; x_1, x_2 \geq 0$; the solution of given problem is

(A) optimal (B) unbounded (C) degenerate (D) infeasible

9. A change in the objective function for a non-basic variable can affect

(A) $z_j - c_j$ values of all non-basic variables
(B) $z_j - c_j$ values of all basic variables
(C) only the $z_j - c_j$ value of that variable
(D) none of these.

10. In Dichotomous Method, let $I_{i-1} = (x_L, x_R)$ and $f(x_1) > f(x_2)$ then I_i is equal to in the maximization $f(x)$ type problem

(A) $I_i = (x_1,\ x_R)$ (B) $I_i = (x_1,\ x_2)$ (C) $I_i = (x_L,\ x_R)$ (D) $I_i = (x_L,\ x_2)$

******END of PART A******

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Part B

Max Marks 30 **Max time: 150 mins** **Date: 23-12-2022**

Note:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q.1 Let $A=\begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}$. For which values of a, rank of the matrix A is less than 3? [6]

Q.2 Find the stationary points of the $f(x_1, x_2) = -x_1^2 + x_1x_2 - x_2^2 + x_1 + 2x_2$. [3]

Q.3 Write the dual of the following LPP:

Maximize $z = -x_1 - 3x_2 - 7x_3 - 5x_4$

subject to

$$-5x_1 + x_2 - x_3 - 4x_4 \leq -10$$
$$-4x_1 + x_2 + 4x_4 \leq -5$$
$$x_1 \geq 0, x_2 \geq 0, \; x_3 \geq 0, x_4 \geq 0$$

Solve the original (primal) problem by dual simplex. [7]

(Show all calculations in tabular form and header of the table should be in textbook format)

Q.4 Consider the LP problem

$$\text{Minimize} \quad z = x_1 - 2x_2 - x_3$$
$$\textit{subject to} \quad x_1 + x_2 + x_3 \leq 6$$
$$x_1 - 2x_2 \leq 4$$
$$x_1, x_2, x_3 \geq 0$$

and optimal table for the given LP is as:

BV	z	x_1	x_2	x_3	x_4	x_5	Solution
z	1	-3	0	-1	-2	0	- 12
x_2	0	1	1	1	1	0	6
x_5	0	3	0	2	2	1	16

where x_4, x_5 are slack variables respectively. If the new constraint $-x_2 + 2x_3 \geq 4$ is added to the given LPP, then use post optimal analysis, to find the solution of new LP problem. [5]

Q.5 Four employees are available to perform four jobs. The time (in minutes) each person to perform each job is given in the following table. Determine the assignment of the employees to jobs that minimizes the total time required to perform the four jobs.

Workers	Jobs			
	Job 1	Job 2	Job 3	Job 4
1	16	32	18	30
2	15	18	20	27
3	24	26	23	28
4	20	16	22	−

Note: Dashes indicate person cannot do that particular job. [5M]

Q.6 Perform 2 iteration to find r_2 and X_2 using the steepest decent method for

$$\text{Minimize } f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

use $X_0 = (0,0)$ as initial point. [4M]

****END****