

1. Find all the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 5 & 3 \end{bmatrix}. \text{ Hence, find the eigen-vector corresponding to the smallest}$$

eigenvalue of the matrix A . [6]

2. By elementary operations (Gauss-Jordan), find inverse of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{bmatrix}. \quad [5]$$

3. Solve the following system by Cramer's rule

$$x_1 + 2x_2 + 3x_3 = 20, \quad 7x_1 + 3x_2 + x_3 = 13, \quad x_1 + 6x_2 + 2x_3 = 0 \quad [5]$$

4. Determine whether the following function is linear transformations or not. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, with $T(x, y) = (x + y, y)$. [4]

5. Determine whether the following set S span \mathbb{R}^3 or not, and hence, determine whether the set S is a basis of \mathbb{R}^3 or not.

$$S = \{ (0, 1, 2), (1, 2, 3), (-1, 0, 1) \}. \quad [4]$$

6. A Biscuit manufacturer has 150 pounds of cherries chocolate and 190 pounds of mints chocolate in stock. He decides to sell them in the form of two different mixtures. One mixture will contain half cherries and half mints by weight and will sell for \$5.00 per pound. The other mixture will contain one-third cherries and two-thirds mints by weight and will sell for \$3.5 per pound. Formulate the problem to maximize his sales revenue. [6]

7. Solve the following LPP by Graphical Method: Maximize $z = x_1 + \frac{1}{2}x_2$

$$\text{Subject to } 3x_1 + 2x_2 \leq 12, \quad 2x_1 \leq 1,$$

$$x_1 + x_2 \geq 8, \quad -x_1 + x_2 \geq 4, \quad x_1, x_2 \geq 0 \quad [5]$$