Birla Institute of Technology and Science, Pilani (Raj.)

First Semester, 2023-24

BITS F218 (General Mathematics III)

Comprehensive Examination (Closed Book)

Max. Marks: 23 **Max. Time: 90 Minutes** **Date: Dec. 13, 2023**

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Note: *Use usual notations and symbols as & when required. Write the answer in the most simplified form and sub-parts of any question should be done together.*

1. Investigate for what values of λ *and* μ the system of linear equation

$$x+2y+3z=4, x+3y+4z=5, x+3y+\lambda z=\mu$$

has (i) unique solution (ii) infinitely many solutions (iii) no solution. **[4]**

2. Determine whether the given set $S=\{t^2+1, t-1, 2t+2\}$ is a basis for the vector space P_2 or not. **[2]**

3. Solve the following system by Cramer's rule

$-x_1+3x_2-2x_3=5,\ 4x_1-x_2-3x_3=-8,\ 2x_1+2x_2-5x_3=7$ **[4]**

4. Find the rank of the matrix

$$A=\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}.$$ **[3]**

5. Consider the following LPP (Primal)

Max $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$$x_1 + x_2 + x_3 = 5,\ x_1 + 4x_2 + x_4 = 9,\ x_1, x_2, x_3, x_4 \geq 0$$

Write the dual of the above problem. **[2]**

6. Solve the following LPP by Big M method.

Minimize $z = -x_1 - x_2$

Subject to $x_1 - x_2 \geq 1,\ 4x_1 + 4x_2 \geq 8,\quad x_1, x_2 \geq 0$ **[5]**

7. Let the LPP be

Maximize $z = 3x_1 + 2x_2 + 5x_3$

and has three constraints.

The optimal table of the above LPP is

Basis	z	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	1	4	0	0	1	2	0	1350
x_2	0	-1/4	1	0	1/2	-1/4	0	100
x_3	0	3/2	0	1	0	1/2	0	230
x_6	0	2	0	0	-2	1	1	20

If a new constraint $3x_1 + 3x_2 + \mathbf{x_3} \leq 600$ is added to the original LPP, what will be solution of the new LPP? [3]
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1. Consider the following LPP

   Min $z = 2x_1 + 3x_2$

   Subject to

   $2x_1 + x_2 \geq 3, \quad x_1 + x_2 = 2, \; x_1, x_2 \geq 0$

   Solve the LPP by Dual Simplex method.

[6]

2. Let the LPP be

   Maximize $z = 4x_1 + 6x_2 + 2x_3$

   with three constraints $x_1 + x_2 + x_3 \leq 3$, $x_1 + 4x_2 + 7x_3 \leq 9$, $x_1, x_2, x_3 \geq 0$.

The optimal table of the above LPP is

Basis	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	0	0	6	10/3	2/3	16
x_1	0	1	0	-1	4/3	-1/3	1
x_2	0	0	1	2	-1/3	1/3	2

If right hand side of the constraints are changed from (3,9) to (9,6), what will be solution of the new LPP? [5]

3. Consider the following transportation problem and find initial basic feasible solution using Vogel's approximation method.

	D1	D2	D3	D4	Availability
S1	5	6	1	0	60
S2	0	2	5	0	50
S3	4	1	2	100	25
Demand	50	15	20	50	

[4]

4. Solve the following Assignment problem for minimize the total cost:

	M1	M2	M3	M4	M5
J1	5	5	***M***	2	6
J2	7	4	2	3	4
J3	9	3	5	***M***	3
J4	7	2	6	7	2
J5	6	5	7	9	1

Where ***M*** is a very large quantity. [4]

5. Mr. George has taken Rs. 10,000 from his father to invest them in a combination of only two stock portfolios with the maximum investment allowed in either portfolio set at Rs. 75,00. The first portfolio has an average return of 10% whereas the second has 20%. In terms of risk factors associated with these portfolios, the first has a risk rating of 4 (on a scale from 0 to 10), and the second has 9. Since he wants to maximize his return, he will not accept an average rate return below 12% or a risk above 6. Hence, he then faces the important question. How much should he invest in each portfolio? Formulate the above as linear programming problem. [3]
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