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Comprehensive Examination Semester I 2019-20

COURSE NO. BITS F314 COURSE TITLE: **Game Theory and its Application**

Date: 03/12/2019 Time: 3 hrs.

5 Questions x 8= **40 Marks** (Total marks)

Question 1:

This question is about a game, called "Deal or no Deal". The monetary unit is M$ (million dollars). The players are a Banker and a contestant. There are 3 cases: 0, 1, and 2. One of the cases contains 1M$ and all the other cases contain zero million dollars. All cases are equally likely to contain the 1 million dollar prize (with probability 1/3). Contestant owns case 0.

Banker offers a price p_0, and contestant accepts or rejects the offer. If she accepts, then Banker buys the content of case 0 for price p_0, ending the game. (Contestant gets p_0 M$ and banker gets the content of the case minus p_0 M$).

If she rejects the offer, then we open case 1, revealing the content to both players. Banker again offers a price p_1, and contestant accepts or rejects the offer. If she accepts, then Banker buys the content of case 0 for price p_1; otherwise we open case 2, and the game ends with contestant owning the content of case 0 and banker owning 0.

The utility of owning x M$ is x for the banker and $x^{1/\alpha}$ for the contestant, where $\alpha > 1$. Assuming α is commonly known, apply backward induction to find a subgame perfect equilibrium. [**8 marks**]

Question 2 (*Part A is compulsory. Attempt* <u>*any one question*</u> *from part B***):**

A. Two division managers can invest time and effort in creating a better working relationship. Each invests $e_i \geq 0$, and if both invest more then both are better off, but it is costly for each manager to invest. In particular, the payoff function for player i from effort levels (e_i, e_j) is $v_i(e_i, e_j) = (a + e_j)e_i - e_i^2$

a. What is the best response correspondence of each player?

b. In what way are the best response correspondences different from those in the Cournot game? Why?

c. Find the Nash equilibrium of this game. [**1+1+2=4 marks**]

B. Imagine a continuum of potential buyers, located on the line segment [0,1], with uniform distribution. (Hence, the "mass" or quantity of buyers in the interval [a,b] is equal to b-a). Imagine two firms, players 1 and 2 who are located at each end of the interval (player 1 at the 0 point and player 2 at the 1 point). Each player *i* can choose its price p_i, and each customer goes to the vendor who offers them the highest value. However, price alone does not determine the value, but distance is important as well. In particular, each buyer who buys the product from player *i* has a net value of v-p_i-d_i where d_i is the distance between the buyer and vendor *i*, and represents the transportation costs of buying from vendor *i*. Thus, buyer $a \in [0,1]$ buys from 1 and not 2 if $v-p_1-d_1>v-p_2-d_2$ and if buying is better than getting zero (Here $d_1=a$ and $d_2=1-a$). Finally assume that cost of production is zero.

Assume that v is very large so that all the customers will be served by at least one firm, and that some customer $x^* \in [0,1]$ is indifferent between the two firms. **What is the best response function of each player.**

OR

What are the pure strategy subgame-perfect Nash equilibria in the bank runs game. How does the bank runs game differ from the Prisoners' Dilemma. [**4 marks**]

Question 3:

A. An employee (player 1) who works for a boss (player 2) can either work (W) or shirk (S), while his boss can either monitor the employee (M) or ignore him (I). Like most employee-boss relationships, if the employee is working then the boss prefers not to monitor, but if the boss is not monitoring then the employee prefers to shirk. The game is represented in the following matrix:

	Player 2		
		M	I
Player 1	W	1,1	1,2
	S	0,2	2,1

a. Write the best response functions of each player.
b. Find the Nash equilibrium of this game.
c. What kind of game does this game remind you of? [**2+1+1=4 marks**]

B. Consider the following situation. An incumbent monopolist decides at date 1 whether to build a small plant or a large plant. At date 2 a potential entrant observes the plant built by the incumbent and decides whether or not to enter. If she does not enter then her profit is 0 while the incumbent's profit is \$25 million with a small plant and \$20 million with a large plant. If the potential entrant decided to enter, she pays a cost of entry equal to \$K million. At date 3 the two firms simultaneously decide whether to produce high output or low output. The profits of the firm are as shown in the following table, where 'L' means 'low output' and 'H' means 'high output' (the figures do not include the cost of entry for the entrant; thus you need to subtract that cost for the entrant); in each cell, the first number is the profit of the entrant (in millions of dollars) and the second is the profit of the incumbent.

If incumbent has small plant

	Incumbent		
		L	H
Entrant	L	10,10	7,7
	H	7,6	4,3

If incumbent has large plant

	Incumbent		
		L	H
Entrant	L	10,7	5,9
	H	7,3	4,5

a. Draw an extensive form game that represents this situation.
b. How many strategies does the potential entrant have? [**2+2=4 marks**]

Question 4:

A.

i) Consider the following game to be played 100 times.

Predator/Prey	Active	Passive
Active	1.7,-0.8	3,-1
Passive	1.6,-0.7	0,0

Which is true about results from backward induction? Why?
{(Passive, Active) can appear in some period; (Passive, Passive) can appear in some period; (Active, Passive) can appear in some period; Only (Active, Active) appears in each period}.

ii. Consider an indefinitely repeated game such that with probability p the game continues to the next period and with prob (1-p) it ends. The "grim trigger'" strategy is such that a player cooperates as long as the other does and defects forever after if the other player defects.

1/2	Cooperate	Defect
Cooperate	4,4	0,5
Defect	5,0	1,1

If the other player uses a grim trigger strategy, **what is the total expected payoff from <u>always cooperating</u>**?

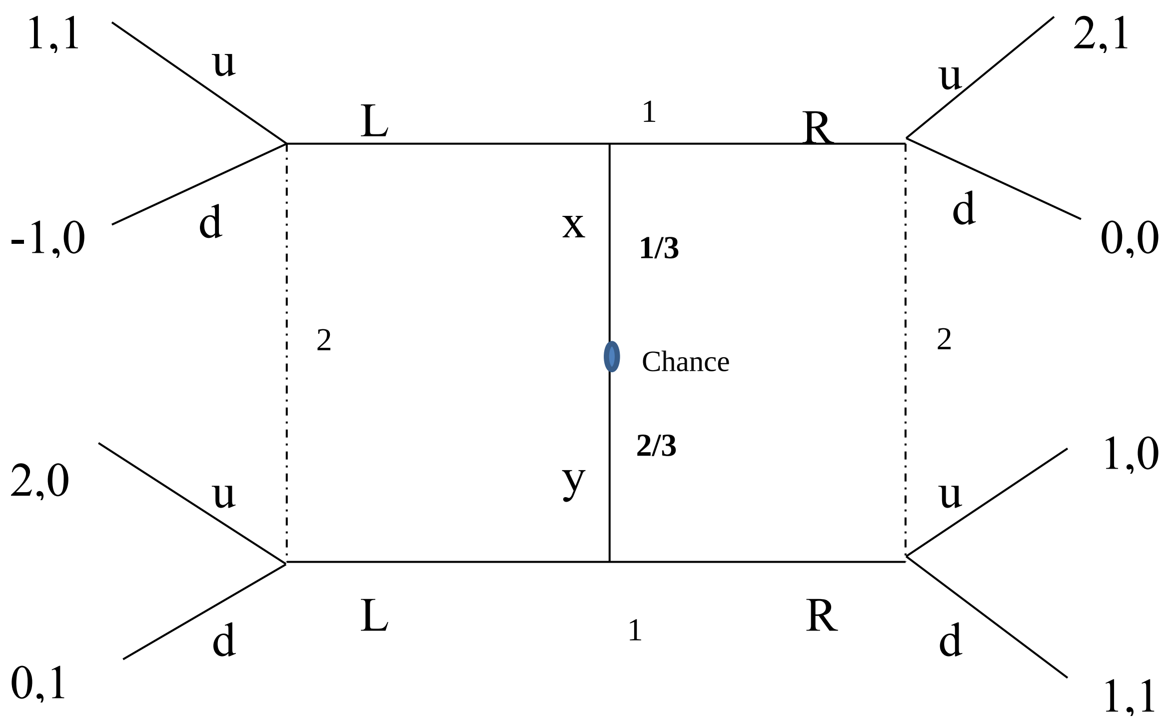
iii. Consider the previous question. Let p^* be the threshold such that when $p \geq p^*$, cooperation is sustainable as a subgame perfect equilibrium by the "grim trigger" strategy. **What is p^***?

[**1+2+2=5 marks**]

B. Consider the sequential bargaining game with a finite number of offers and where the two players take turns making offers about how to divide a pie of size one. Show that in the case of N=2 offer, the unique sub-game perfect equilibrium involves an immediate $(1-\delta_2, \delta_2)$ spilt.

[**3 marks**]

Question 5:
Consider the signaling game given in figure below:



Describe all the pure strategy **<u>pooling</u>** **and** **<u>separating</u>** **perfect Bayesian equilibria in the above signaling game.** [**8 marks**]