Birla Institute of Technology & Science, Pilani, Pilani Campus – Rajasthan
Mid-Sem (Closed Book) BITS F 314 [Game Theory & Applications]

Maximum Marks: 60

Time Duration: 90 Minutes
Dated: 1/Nov/2022

Instructions:
- Read the questions thoroughly before answering. All questions are compulsory. Start each question on a new page.
- Calculation(s) to arrive at the result(s) and its Interpretation are necessary to get marks.
- Calculator is allowed.
- Make sure that you have correctly mentioned your Name, ID, Course, and other details on your answer sheet.

Question 1: Two butter-selling firms namely, Amul and Brittania, are competing in the Pilani market. Both can charge either a low price of INR 20 per packet or a high price of INR 40. The costs for the two firms are the same at INR 10000. With a lower price, the firms can sell a total of 1000 packets as compared to 500 packets at a higher price. If both charge the same price, they split the market equally. If one charges low and another high, then the firm that is charging less gets the entire market and the other cannot sell anything.

a) Given the above market scenario, construct the normal form of the game with profits as payoffs and find out the Nash Equilibrium in a pure form.

b) Find out the equilibrium outcome of this game under the assumption that the preferable strategy for both firms is to play a secure strategy. Compare this outcome with the NE.

[5+5]

Question 2: In a war between Russia and Ukraine, if Russia attacks, Ukraine can fight resulting in payoffs of (-)1 for both, or retreat, resulting in a payoff of 5 to Russia and (-)3 to Ukraine. If Russia does not attack, Ukraine will either attack, resulting in a payoff of 2 to Ukraine and (-)2 to Russia, or Ukraine will not attack, resulting in a payoff of 10 to both.

a) Describe the above war scenario using a game tree form by carefully labeling all of its components.

b) Find out all subgame perfect nash equilibria of the game.

c) Describe the game in a simple strategic form and find a pure strategy NE of this game that is not subgame perfect.

[5+3+7]

Question 3: Rohan has invited Chetan to his birthday party. Rohan must choose whether to hire a DJ or not. Simultaneously, Chetan must decide whether or not to go to the party. Chetan is a good friend of Rohan but hates loud music at parties. He even hates other people having fun with the loud music played by DJ.

Chetan's payoff from going to the party is 4 if there is no DJ and 0 if there is a DJ. However, Chetan's payoff from not going to the party is 3 if there is no DJ and 1 if there is a DJ. On the other hand, Rohan likes DJ and especially wants to see Chetan's reaction to his decision of hiring DJ. Rohan also does not likes to pay to DJ. Rohan's payoff if Chetan comes to the party is 4 if there is no DJ, but (8-*x*) if there is a DJ. Where *x* is the cost of hiring a DJ. Rohan's payoff if Chetan does not come to the party is 2 if there is no DJ, but (3-*x*) if there is a DJ.

   a) Write down the normal strategic form of the game between Rohan and Chetan.

   b) Suppose *x=0*, Identify any dominated strategies and explain. Also, find NE and write down the equilibrium strategy profiles and payoffs.

   c) Suppose *x=2*, Identify any dominated strategies and explain. Also, find NE and write down the equilibrium strategy profiles and payoffs.

$$[2+5+8]$$

Question 4: Demand for milk can be characterized by $Q = 100 - P$, where Q is in thousands of packets of milk sold and P is the price per packet. There are two firms producing milk packets, Saras (*S*) and Amul (*A*), with identical cost conditions as follows:

$$C_i = 10Q_i + \frac{1}{2}Q_i^2 \qquad\qquad i = S \text{ and } A \qquad\qquad Q = Q_S + Q_A$$

   a) The top management in both firms independently recognizes the oligopolistic nature of the milk industry and plays Cournot. How much output is to be produced by each producer and what price is to be charged in the market? Also, calculate each producer's profits.

   b) What if the two firms collude and make a cartel? How much output is to be produced by each producer and what price is to be charged in the market? Also, calculate each producer's profits.

   c) The managers of these firms realize that explicit agreements to collude are illegal. Each firm must decide on its own whether to produce the Cournot quantity or the cartel quantity. To aid in making the decision, managers of both firms construct a payoff matrix and decide between the available options. Construct a payoff matrix that helps the managers to take the final decision. Given this payoff matrix, what output strategy is each firm likely to pursue? Justify your answer with proper reasoning.

$$[5+7+8]$$

*****All the Best*****