BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
FIRST SEMESTER 2023-24
Mid-Semester Exam (Closed Book)

Course No.	**: BITS F314**	Max. Marks	**: 60**
Course Title	**: GAME THEORY & ITS APPLICATIONS**	Duration	**: 90 Minutes**
Date	**: 09/Oct/2023**		

Instructions:

- All the questions are compulsory. All parts of the questions should be answered sequentially and together.
- Show all the necessary calculations wherever required.
- Please use a pen only while answering the question and drawing the graphs.
- Overwritten/ambiguous answers will not be evaluated. The exchange of calculators is not permitted.
- While calculating, use two points after decimal without rounding off.

Q1: Market demand for edible oil can be characterized by $Q = 200 - 2P$, where Q is the total quantity sold in thousands of packets, which is the sum of the units sold by individual firms ($Q = Q_s + Q_F$) in the edible oil industry, and P is the price per packet. Two firms, Saffola and Fortune, produce edible oil with identical cost conditions as follows: $C_i = 8Q_i + \frac{1}{2}Q_i^2 \quad (i = S, F)$. Given this information, answer the following questions.

(a) The top management in Saffola and Fortune independently recognizes the oligopolistic nature of the edible oil industry and plays Cournot (non-collusive). Based on this information, derive the reaction function of these two firms and identify the Nash equilibrium. How much output will both the firms produce, and what price will be charged in the market? Also, calculate each firm's profits.

(b) Considering the market conditions, suppose the two firms collude and make a cartel. How much output will each firm produce, and what price will the market charge? Also, calculate each firm's profits.

(c) The managers of these firms realize that explicit agreements to collude are illegal. Each firm must decide on its own whether to produce the Cournot or cartel quantity. To aid in making the decision, managers of both firms construct a payoff matrix and choose between the available options. Construct a payoff matrix that helps the managers to make the final decision. Given this payoff matrix, what output strategy (dominant strategy) is each firm likely to pursue in the Nash Equilibrium? **[9+4+7=20M]**

Q2: Suppose there are two firms, Pears and Dove, are producing differentiated Soaps having the following cost functions: $C(Q_P) = 10Q_P$ and $C(Q_D) = 5Q_D$. The demand functions for these firms are given as:

$$Q_P(P_P, P_D) = 1000 - 20P_P + 15P_D$$
$$Q_D(P_P, P_D) = 800 + 5P_P - 15P_D$$

a) Based on the above information, using the Bertrand model, compute the best response function for both firms.

b) Find the Bertrand equilibrium price and profit for both firms. **[4+6=10M]**

Q3: Consider an attacker's and a defender's strategies in a penalty stroke round, determining the Women's Hockey World Cup 2022 winner. The following payoff matrix represents the attacker (in rows), who must choose whether to aim Left, Right, or Center, and the defender (in columns), who must dive to the attacker's Left, Right, or Center.

		Defender		
		Left	**Right**	**Center**
Attacker	**Left**	0.65, 0.35	0.95, 0.05	0.95, 0.05
	Right	0.95, 0.05	0.65, 0.35	0.95, 0.05
	Center	0.95, 0.05	0.95, 0.05	0, 1

The payoffs in each cell represent the probability that the attacker scores. For instance, if the attacker shoots to the right and the defender guesses the correct way (i.e., she dives to the right), the probability that the attacker will score is 0.65, while the defender will save is 0.35. Based on this information, answer the following questions.

(a) Find the pure strategy Nash equilibrium of this game.

(b) Find the mixed strategy Nash equilibrium of this game.

(c) What is the probability that a goal is scored? **[2+10+3=15M]**

Q4: Please read the questions carefully related to voting and answer them accordingly:

a) Define the Median voter concept in the Hoteling electoral competition model, identify Nash Equilibrium conditions from the best response functions, and show the graphical representation of the median voter model.

b) Consider the following example (*non-strategic game*): There are 55 voters in a locality, and five candidates (A, B, C, D & E) are contesting for a representative position. The preference pattern of these voters for the given candidates is: 18 voters prefer A > D > E > C > B; 12 voters prefer B > E > D > C > A; 10 voters prefer C > B > E > D > A; 9 voters prefer D > C > E > B > A; 4 voters prefer E > B > D > C > A; and 2 voters prefer E > C > D > B > A.

Given this information, identify the winners using the:

i) Plurality rule
ii) Borda count (rankings: first to last as four to zero)
iii) Instant Run-off voting

[*Please note that you need to mention all the steps in each scenario to identify the winner while answering. Merely writing the winner will not be considered*] **[9+6=15 M]**