Name_____ **ID**_____

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
FIRST SEMESTER 2023-2024
Comprehensive Examination Part-A & B (Closed Book)

Course No.	**: BITS F314**	Maximum Marks	**: 40**
Course Title	**: Game Theory & Its Applications**	Duration (Max)	**: 90 Minutes**
Date	**: 6/Dec/2023**	Weightage	**: 20%**

Instructions:

- Write your ***Name*** and ***ID Number*** clearly on the answer sheet. There are two parts: Part A & B.
- Part A carries 15 multiple-choice questions. Answers to questions in Part A should be answered in the given space on this page only. There will be no negative marking.
- Part B contains five short answer-type questions. You will be given one supplementary answer book to write your answers.
- Do not overwrite. Use of pencil is not allowed. Do not use a red color pen.
- At the end, submit both the question paper and the answer sheet.
- A calculator is allowed; however, the exchange of a calculator is not permitted.

PART A: Space to answer MCQs										
Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Answer										
Question	**11**	**12**	**13**	**14**	**15**	**Part A - Total Marks**		**Part A & B – Total Marks**		
Answer										

Part A: Multiple Choice Questions (1 Mark Each)

Q1: Which of the following circumstances will result in a Nash equilibrium?

A) All players have a dominated strategy and each player chooses its dominated strategy
B) All players have a dominated strategy, but only some choose to follow it
C) All players have a dominant strategy, and none choose it
D) None of the above is correct

Q2: When a single strategy profile stands out from among multiple Nash equilibria because the players share a common understanding of the environment in which the game is being played, this is called as:

A) Coordination equilibrium point B) Focal point equilibrium
C) Dominated point equilibrium D) Definite point equilibrium

Q3: An auction in which a seller begins by offering an item for sale at a relatively high price and then reduces the price by fixed amounts until receiving a bid is known as a:

A) Dutch auction B) English auction C) second-price auction D) sealed-bid auction.

Q4: Viewed from the perspective of the Stackelberg model, the Cournot solution is not a Nash equilibrium because

A) each firm is not maximizing profits given the other's output
B) each firm has an incentive to take advantage of knowledge of its rival's reaction function
C) quantity supplied is not equal to quantity demanded at the prevailing price
D) it is not a perfectly competitive outcome

Q5: A "credible" threat is a threat of action that a player

A) makes in a believable way B) commits himself or herself to carry out.
C) cannot retract D) would be willing to undertake if in a position to do so

Q6: A strategy adopted by one player in response to an unanticipated move by a rival known as,

A) Trigger strategy B) Dominated strategy C) Optimum strategy D) Credible strategy

Q7: Paes, a star tennis player, is playing with the number one player in the world, Bhupathi. Before the match, Paes decided that he would serve 20 percent of his serves to Bhupathi's backhand, 30 percent of his serves to Bhupathi's forehand, and 50 percent of his serves straight at Bhupathi. In the language of game theory, this is known as:

A) a pure strategy B) a dominant strategy C) a mixed strategy D) a maximin strategy.

Q8: Consider the static pricing game depicted in the following Table. Suppose that the discount rate in this infinitely repeated game is 5 percent. Given this information estimate, what is the present value of the stream of payoffs to each firm from a no-collusion strategy?

	Firm B – High Price	**Firm B – Low Price**
Firm A – High Price	1 million, 1 million	100000, 5 million
Firm A – Low Price	5 million, 100000	250000, 250000

A) 5250000 B) 5520000 C) 2550000 D) 5260000

Q9: For infinitely repeated games in which the players follow a tit-for-tat strategy, which one of the following outcomes is NOT possible?

A) The players cooperate with one another until someone decides to not cooperate, and then the other players will not cooperate for some period of time

B) There can be dominant strategies

C) If the information about another player's action is limited, then some cooperative actions may be incorrectly interpreted as "not cooperate"

D) All of the above are possible outcomes.

Q10: Consider the following game in which two firms decide how much of a homogeneous good to produce. The annual profit payoffs for each firm are stated in the cell of the game matrix, and Firm A's payoffs appear first in the payoff pairs. What are the dominant strategies in this game?

	Firm B - low output	**Firm B - high output**
Firm A - low output	300, 250	200, 100
Firm A - high output	200, 75	75, 50

A) Both firms produce low levels of output B) Both firms produce high levels of output

C) Firm A's dominant strategy is to produce low levels of output, but Firm B does not have a dominant strategy.

D) Firm B's dominant strategy is to produce low levels of output, but Firm A does not have a dominant strategy.

Q11: What is the dominant strategy for bidders in an English oral auction?

A) Keep bidding until all other bidders quit, regardless of your reservation price.

B) Bid until the previous bid price equals the reservation price of the last bidder.

C) Bid until the first-price and second-price bids are equal.

D) Stop bidding once the price exceeds your reservation price.

Q12: Consider the following game that represents the payoffs from different advertising campaigns (low, medium, and high spending) for two political candidates that are running for a particular office. The values in the payoff matrix represent the share of the popular vote earned by each candidate. Under the following version of the game with simultaneous moves, what is the Nash equilibrium?

	Candidate B - low	**Candidate B - medium**	**Candidate B - high**
Candidate A - low	50, 50	40, 60	20, 80
Candidate A - medium	60, 40	50, 50	35, 65
Candidate A - high	80, 20	65, 35	50, 50

A) Neither candidate has a dominant strategy, but the Nash equilibrium occurs where both candidates use medium advertising campaigns.

B) Candidate A's dominant strategy is high, Candidate B's dominant strategy is high, and this is the Nash equilibrium.

C) Neither candidate has a dominant strategy, but the Nash equilibrium occurs where both candidates use high advertising campaigns.

D) This simultaneous game has no Nash equilibrium (in pure strategies).

Q13: Consider two firms, Aqualik and Bisleri, operating at a constant marginal cost of ₹ 10. The market demand for their product is Q = 130 – P. Aqualik Firm can choose its output before Bisleri Firm. Bisleri Firm chooses its output after observing Aqualik's choice. How much does being a first mover increase Aqualik's profits compared to the Cournot duopoly profit?

A) ₹ 0 B) ₹ 200 C) ₹ 400 D) ₹ 600

Q14: Each rule in …….. requires the injurer to compensate the victim for a loss if and only if both the victim is sufficiently careful and the injurer is sufficiently careless; the required compensation is the total loss.

A) negligence with contributory negligence B) Strict liability
C) Mixed response D) Simultaneity action

Q15: What happens to the market outcome if cartel members cheat on the collusive agreement?

A) Price declines, but firm-level quantities remain the same because the firms act like price-takers
B) Price and quantity revert to the single-seller monopoly equilibrium outcome
C) Other firms raise prices so that the average market price remains unchanged
D) Price declines and quantity increases toward the perfectly competitive equilibrium

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Part B
Short Answer Type Questions
(Write your answer in the Supplementary Answer Book)

Q16: Describe the war of attrition in a game theoretic framework. What is the Nash equilibrium in the war of attrition game? Present it graphically. What are its properties? **[5M]**

Q17: Assume that there are four participants, namely Hari, Krishna, Shyam, and Murali, who are bidding for an ancient artwork. Moreover, each bidder is aware of their own assessment of the artwork but remains unaware of the other bidder's valuations. The values placed on the ancient artwork by Hari, Krishna, Shyam, and Murali are VH=₹ 450, VK=₹ 370, VS=₹ 325 and VM=₹ 295. Based on the opinion of disinterested experts, the lowest possible value of the artwork is L= ₹ 150 and the highest possible value is H = ₹ 350. Based on this information, answer the following questions.

a) If player valuations are independent, random, and uniformly distributed, how much should each risk-neutral player bid (Optimum bid) for the ancient artwork in a sealed-bid first-price auction game?
b) If this is an English auction, which player wins?

[5M]

Q18: Suppose you own a sports showroom and must decide at what price to sell popular new football shoes. You know that your competitor across the street is selling the same shoes, so you must consider the price at which others are selling to ensure you attract as many customers as possible. In addition to this, you also have the following information:

- If you and your competitor sell the shoes at ₹50, you will sell 55 % of the total number of shoes sold between you.
- If you sell the shoes at ₹50, but your competitor sells the shoes at ₹70, you will sell 70% of the total number of shoes sold between you.
- If you sell the shoes at ₹70, but your competitor sells the shoes at ₹50, you will sell 40% of the total number of shoes sold between you.
- If you and your competitor sell the shoes at ₹70, you will sell 55% of the total number of shoes sold between you.

a) Represent the above information as a game and answer the question: If your and your competitor's primary goal is to sell as many shoes as possible, what is the best price to sell the shoes at?
b) After doing research, you find that 100 people will be buying these shoes. If you and your competitor's primary goal is to maximize the amount of money made from selling the shoes, what is the best price to sell the shoes at?

[5M]

Q19: Two students, Lily and Mily, must write their First Degree thesis (FDTS) to complete their degree. They need to choose a professor to supervise them for the thesis. They have to choose the professor independently and without knowing others' selections. Three professors are available for the role of supervisor. Prof. Sharma, Prof. Verma, and Prof. Arora. The utility of a student is given by the amount of help she receives from her supervisor, which is quantified as 40 for Prof. Sharma, 60 for Prof. Verma, and 50 for Prof. Arora. However, if the two students select the same professor as their supervisor, they only get 70% of the utility that they would get if the professor had only one of them to supervise. Given this information:

a) Write the game in a normal form
b) How many Nash equilibria does this game have? Calculate them. **[5M]**

Q20: The market demand for Idli in a food court is Q = 200-10P. Four stalls sell Idlis in the food court. Their marginal cost equals the average cost and is identical at ₹10. The stall with the lower Idli price will receive all the business, as there is no customers' loyalty in this food court. If all the stalls set the same price, they will split the business equally.

a) If all the stalls engage in price competition in a one-period game, what is the equilibrium price and quantity of Idlis sold in each stall in the food court?
b) If the four stalls agree to collude and the market demand remains the same, what is the equilibrium price and quantity of Idlis sold per stall in the food court? **[5M]**

*******All the Best*******

Birla Institute of Technology & Science, Pilani, Pilani Campus – Rajasthan
Comprehensive Exam (Open Book)
BITS F 314 [Game Theory & Applications]

Maximum Marks: 40 **Time Duration (Max)**: 90 Minutes
Session 2023-24 (I) **Dated**: 6-Dec-2023
Instructions:

- Read the questions thoroughly before answering. All questions are compulsory. Start each question on a new page. Calculation(s) to arrive at the result(s) and its Interpretation are necessary to get marks.
- The calculator is allowed, but the exchange of the calculator is not.
- Ensure you correctly mention your Name, ID, Course, and other details on your answer sheet.
- During calculation, please consider two points after decimal without round-off.

Part C
Long Answer Type Question

Q1: Reena and Ram, the top gamblers in Finland, decided to provide consultancy services to new gamblers. Reena first needs to decide whether to open an office in Finland or to move to Singapore. If she moves to Singapore, both will earn 5 million dollars during the next twenty years. Ram must stay in Finland, and if Reena also stays, he will decide whether to cooperate with Reena or not. If Reena stays, she will observe Ram's decision, and then she will have to decide whether to cooperate or not.

- If Reena stays, their earning over the next twenty years will be 6 million dollars to both if they both cooperate,
- 2 million dollars to both if neither cooperates
- 0 to Ram, and 4 million dollars to Reena if Ram cooperates and Reena does not,
- 0 to Reena and 4 million dollars to Ram if Reena cooperates and Ram does not.

Given this information, answer the following questions:

a) Describe the game using a tree and label all its components as per the class discussion.
b) Solve the game using the backward induction method.
c) Describe the game in strategic form, find all its pure-strategy Nash equilibria and indicate which of these is subgame perfect. **[3+2+3]**

Q2: Kataria has decided to introduce a revolutionary video game. As the first firm in the market, it will have a monopoly position for at least some time. In deciding what type of technology is in production, it has the choice of two technologies:

- Technology *A* is publicly available and will result in annual costs of: $C^A(q) = 20 + 8q$
- Technology *B* is a proprietary technology developed in Kataria's research labs. It involves a higher fixed cost of production but lower marginal costs: $C^B(q) = 70 + 2q$

Kataria must decide which technology to adopt. Market demand for the new product is $P = 30 - Q$, where Q is the total industry output.

a) Suppose Kataria was certain that it would maintain its monopoly position in the market for the entire product lifespan (about five years) without the threat of entry. Which technology would you advise Kataria to adopt? What would be Kataria's profit given this choice?
b) Suppose Kataria expects its main opponent, Salaria, to consider entering the market shortly after Kataria introduces its new product. Salaria will have access only to Technology *A*. If Salaria does enter the market, the two firms will play a Cournot game (in quantities) and arrive at the Cournot-Nash equilibrium.
  i) If Kataria adopts Technology *A* and Salaria enters the market, what will be the profit of each firm? Would Salaria choose to enter the market given these profits?
  ii) If Kataria adopts Technology *B* and Salaria enters the market, what will be the profit of each firm? Would Salaria choose to enter the market given these profits?
  iii) Which technology would you advise Kataria to adopt given the threat of possible entry? What will be Kataria's profit given this choice?

[5+10]

Birla Institute of Technology & Science, Pilani, Pilani Campus – Rajasthan
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BITS F 314 [Game Theory & Applications]

Q3: Two heroes, John and Salman, enter the snake island in Andaman to save a princess. The King has promised a reward of 20 gold coins to the savior (s) of his daughter. Both heroes reach the island, where the snake waits in front of the pole where the princess is tied. As they enter the room, the snake makes an evil laugh and sets the place in flames. The two heroes must decide what to do on the spot and cannot consult each other. Attack (A) the snake or save (S) the princess? They are strong enough to kill the snake, even if fighting alone. Also, each of them is skilled enough to save the princess.

- However, if they both attack the snake, they will quickly get rid of it, but the princess will die. It means no reward, i.e., utility 0 for both.
- If they both attempt to save the princess, it is even worse, as the snake is left undisturbed and able to kill them, which means a utility of (-) 100 for both.
- Their mission is accomplished if they manage to kill the snake and save the princess.

However, before the game ends, a last stage can now be played. The hero who killed the snake has a further chance to play. He will have the power of a snake; with that help, he can kill (K) another hero on the spot who is busy saving the princess. This way, the reward is not split among the players, and the dead hero gets utility (-)100. Of course, the hero who has this opportunity can also turn it down by being a fair (F) friend to his friend. This way, the reward is split between them.

John considers it disgusting to kill his friend to get a bloody reward. He attributes utility (-) 20 to this outcome. Salman is not so uptight about morals. Getting the entire reward gives him utility 20. Splitting the reward gives utility 10 to both. Both heroes are perfectly rational and fully informed about all the details described above. Given this information:

a) Write down the extensive form of this game.
b) Apply backward induction to decrease one level in the game. Write down the normal form of this reduced version of the game.
c) Both players have two binary choices (the second one may not happen but must be planned anyway), so they have four strategies as pairs of actions. For example, SF means I try saving the princess first, and if given the opportunity of betraying my ally, I will not." What kind of outcome is (AK, AK)? **[4+4+4]**

Q4: Two firms, Birla and Tata, produce heterogeneous goods. If the two firms set prices P1 and P2, respectively, the quantities demanded of the products of the two firms will be:

$$q_1(p_1, p_2) = 140 - p_1 - \left(p_1 - \frac{p_1 + p_2}{2}\right)$$
$$q_2(p_1, p_2) = 140 - p_2 - \left(p_2 - \frac{p_1 + p_2}{2}\right)$$

Suppose the two firms have constant marginal costs, $MC_1 = 40$ and $MC_2 = 40$, and that they have to set their prices simultaneously. What is the Nash equilibrium? What are the Nash equilibrium profits for Birla and Tata? **[5]**

*********All the Best*******