

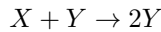
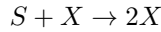
BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
 SECOND SEMESTER 2021-22
 BITS F316: *Nonlinear Dynamics and Chaos*
 Midterm test (Part B)
 Open Book

Total marks: 40

Time: 60 mins

1. *Oscillation in equilibrium chemistry !*

The following chemical sequence from a substrate to a product $S \rightarrow P$ is given by



If k_1 , k_2 and k_3 denote the rate constants for the three reactions respectively, the rate equations for the concentrations $x = [X]$, $y = [Y]$ of the two intermediates X and Y are given by

$$\frac{dx}{dt} = (k_1[S] - k_2y)x$$

$$\frac{dy}{dt} = (k_2x - k_3)y$$

- (a) What are the equilibrium concentrations (x_0, y_0) of $[X]$ and $[Y]$ for a fixed substrate concentration $[S]$?
- (b) Discuss the stability of the fixed point and show that the concentration of the intermediates oscillate ! [2 + 6]

2. Consider a 3 dimensional dynamical system

$$\dot{x} = x + y^4, \quad \dot{y} = -y, \quad \dot{z} = -2z + y^3$$

- (a) Is the origin $(0, 0, 0)$ a Hyperbolic fixed point?
- (b) What is the 3×3 linearized stability matrix A ?
- (c) Find the spaces E_U and E_S .
- (d) What is the behaviour of the flow in the $x - y$ plane around the origin?
- (e) What is the stable manifold W_S for this system? Is the relation between E_S and W_S for this system consistent with the stable-manifold theorem ? [1 + 2 + 2 + 1 + 2]

3. Consider the celebrated Lorenz system

$$\dot{x} = \sigma(y - x), \quad \dot{y} = rx - y - xz, \quad \dot{z} = xy - bz$$

where r, σ, b are positive constants. What can be said about the stability of the origin from linear theory and from Lyapunov theory assuming a Lyapunov function $L = \alpha x^2 + \beta y^2 + \gamma z^2$ (assume $0 < r < 1$). [6]

4. Consider a dynamical system which may be written using the complex variable $z = x + iy$ as

$$\dot{z} = (z - 3i)(z + 3i)$$

What are the fixed points and their stabilities? Draw a generic phase flow in the (x, y) plane and find the winding number for a closed contour which includes all the fixed points in its interior. [6]

5. For the following systems identify the fixed points/ limit cycles (if any) and discuss their stability

(a)

$$\dot{x} = -y + f(r)x, \quad \dot{y} = x + f(r)y \quad \text{where } r = \sqrt{x^2 + y^2}$$

(b)

$$\dot{r} = r^3 - 3r^2 + 2r, \quad \dot{\theta} = 1$$

(c)

$$\dot{\mathbf{r}} = \mathbf{r}(\alpha \sin \beta r^2)/r^2 + \gamma (\hat{x}j - \hat{y}i)$$

where α, β, γ are positive constants and $\mathbf{r} = x\hat{i} - y\hat{j}$ and $r = |\mathbf{r}|$

[4 + 4 + 4]