Q1.: (a) Discuss the dynamics of $\dot{x} = r \ln x + x - 1$ near x=1, and shows that the system undergoes a transcritical bifurcation at a certain value of r. Find new values of X and R such that this system reduces to the prototype equation used for the transcritical bifurcation. (5)

(b) A model is proposed for the budworm population dynamics as:

$$\dot{N} = RN \left(1 - \frac{N}{K}\right) - \frac{BN^2}{A^2 + N^2}$$

Where $N(t)$ represents the budworm population, R as growth rate, K as a carrying capacity and parameters $A, B > 0$. The first part of the equation is logistic and the second one is predation. Convert it to the dimensionless form in such a way that the dimensionless groups are pushed into the logistic part of the equation only. (3)

Q2.: (i) Discuss the Ghosts and Bottlenecks aspects in the non-uniform oscillator: $\dot{\theta} = \omega - a \sin \theta$, symbols here have their usual meanings. (3)

(ii) Discuss the homoclinic and heteroclinic orbits. (2)

(iii) A phase portrait of a two dimensional system is given at the end of the page.

(a) What type of system is it on time reversal aspect and why? (1)
(b) Discuss the stabilities of all the fixed points given in the portrait. (2)
(c) Comments on whether the given system is conservative or not (with proper arguments).
(2)

Q3.: Consider a two dimensional system as: $\dot{x} = 5x + 2y$; $\dot{y} = -17x - 5y$.

(a) Find the eigen values of the system.
(b) Find the fixed points and analyze their stabilities.
(c) Draw the phase portrait of the system. (2+3+2)

Q4.: Consider the circuit equation $L \ddot{Q} + R \dot{Q} + \frac{Q}{C} = 0$, where $L, C > 0$ and $R \geq 0$.

(a) Rewrite the equation as a two-dimensional linear system.
(b) Classify the fixed point at the origin, based on the value of $R^2 C - 4L$ is positive, negative, or zero. Sketch the phase portrait in all three cases. (1+4)