BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

SECOND SEMESTER 2022-2023

Comprehensive Exam

Part-I (Open Book)

Course No: BITS F316 Course Title: Nonlinear Dynamics and Chaos

Date: 20.05.2023

Suggested. Time: 45 min. Total Marks: 10

Note: Use Box to answer wherever asked.

Name:	ID No.:

Q1. Find out the stable periods for the logistic map $x_{n+1} = ax_n(1 - x_n)$ for a=3.2. (1 marks)

Q2. Which of the following statements is/are true for Henon map (tick it/them)? (0.5 marks)

(A) It is a non-invertible nonlinear map.
(B) It is an invertible nonlinear map.
(C) It is dissipative in nature for $|b| > 1$.
(D) It is dissipative in nature for $|b| < 1$.

Q3. Which of the following statements is/are false for Lorenz system (tick it/them)? (0.5 marks)

(A) It is a non-autonomous nonlinear system of 3 dimensions.
(B) It is an autonomous nonlinear system of 3 dimensions.
(C) It is a conservative system.
(D) The strange attractor of Lorenz system is topologically equivalent to the product of cantor set and ribbons.

Q4. Consider a Non-uniform Oscillator $\dot{\theta} = 1 - a \sin \theta$. The time period of the oscillation is observed to be 4π second. Find the value of a. (1 marks)

Q5 The even seventh Cantor set is constructed as follows: divide [0,1] into seven equal pieces: delete pieces 2, 4 and 6; and repeat on sub-intervals. Find out the similarity dimension of the set. (1 marks)

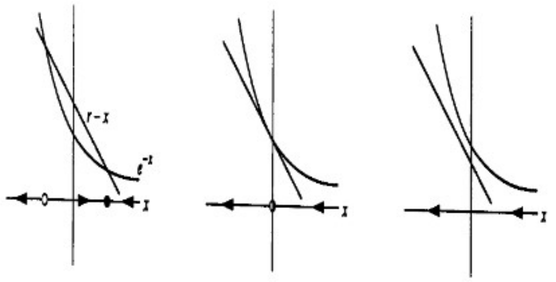
Q6. For any nonlinear map, the first bifurcation from period 1 to period 2 occurs at a_1= 0.3675 for the control parameter a. The next period doubling bifurcation (period 2 to period 4) occurs at a_2= 0.905. Find out the approximate value of parameter a (a_4) at which you have period 8 to period 16 bifurcation

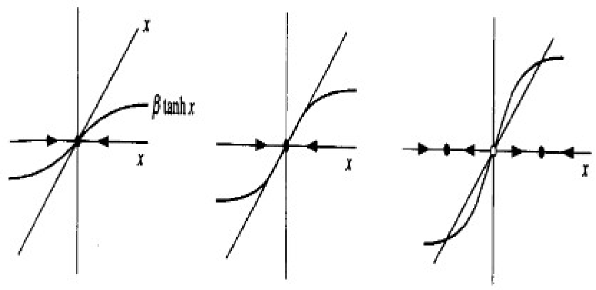
(1.5 marks)

Q7. The Jacobian Matrix A for the two dimensional system for the fixed point (1,1) is given as $A = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$. Find the eigen value and draw the phase portrait near (1,1). ((1+1) marks)

Q8. Below 2 figures represent some sort of bifurcation. Name the bifurcation involved in each figure.

(1 marks)





Q9. Describe the chaotic behavior in 4 lines (the answer should be concise and focused). (1.5 marks) marks)

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

SECOND SEMESTER 2022-2023

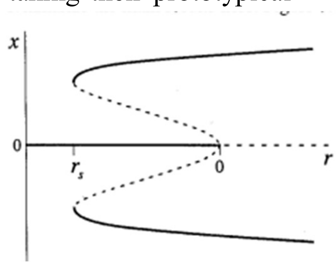
Comprehensive Exam

Part-II (Closed Book)

Course No: BITS F316 Course Title: Nonlinear Dynamics and Chaos Date: 20.05.2023

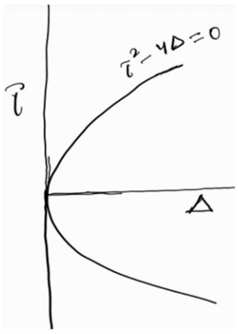
Max. Time: 135 min. Total Marks: 30

Q1.: (a) Discuss the Saddle-Node and Supercritical Pitchfork Bifurcations by taking their prototypical examples of the one dimensional first order system by varying their control parameters. (3)



(b) The bifurcation diagram for the Subcritical bifurcation occurred in the system $\dot{x} = rx + x^3 - x^5$ is shown in the given figure. Discuss the jumps and hysteresis phenomena as the parameter r is varied. (2)

Q2.: (a) The type and stability of all the fixed point can be shown from the given diagram whose axes are the trace τ and the determinant Δ of the matrix A. Identify different regions in the diagram representing different types of fixed points. Draw the clear diagram while answering. (2)

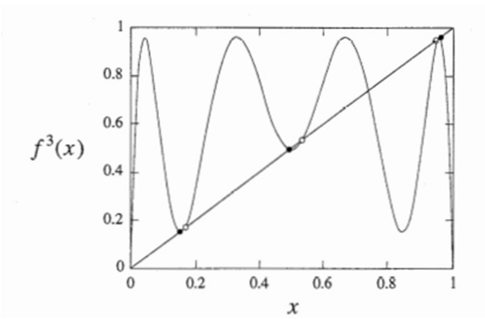


(b) Consider a simple pendulum $\ddot{\theta} + \sin\theta$.

(i) Convert it into two first order equations and find the fixed points. (2)

(ii) Analyze their stabilities and draw a schematic sketch of the phase space. (2)

Q3,: (a) We observed an order in chaos while changing the control parameter r in the one dimensional logistic map. The period-3 behaviour is observed for $3.8284 \ldots \leq r \leq 3.8415 \ldots..$ The third iterate map $f^3(x)$ is shown in the given figure. Analyze the stabilities of the fixed points of the third iterate map $f^3(x)$ from the figure with proper argument. (2)



(b) Define the Tent map. From the calculation of Liapunov exponent, find the range of the parameter of the tent map for which it will have chaotic solution. (3)

(c) Define superstable cycle of period p in the one dimensional map (say logistic map) and find its Liapunov exponent. (2)

Q4: (a) Write down the equations of the Lorenz system. (1)

(b) The Duffing Oscillator is defined as $\ddot{x} + \delta\dot{x} - x + x^3 = F\cos\omega t$, where $\delta > 0$ is the damping constant.

(i) What type of this system is it from the perspective of the time dependence? (1)

(ii) Convert it to a set of the first order equations. (1)

(iii) Find the potential of the system and plot it. (3)

Q5.: (a) Define the baker's map which maps itself in the square $0 \leq x \leq 1,\ 0 \leq y \leq 1$. (1.5)

(b) Find the box dimension of the baker's map. (2)

(c) Discuss the self-similarity aspects in the Henon map. (1.5)

(d) Consider the logistic map $x_{n+1} = r\,x_n\,(1 - x_n)$at the parameter value $r = r_{\infty =}$ 3.5699456 ..., corresponding to the onset of chaos. Discuss that the attractor at this parameter is a Cantor like set. (1)