BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
First Semester, 2023-24
BITS F343 (Fuzzy Logic & Applications)
Comprehensive Examination (Regular, Open Book)

Max. Time: 180 minutes Date & Time: Monday, December 18, 2023, 9:00 AM-12:00 Noon Max. Marks: 90

Note: The notations have usual meaning as and when required. Use $A, \widetilde{A}, \mu_{\widetilde{A}}(x)$ for crisp set, fuzzy set and membership grade function respectively. Do all sub-parts together. Start new question from fresh page.

1. Consider the fuzzy logic environmental controller with following two inputs (Temperature, humidity) and one output (speed) under the defined fuzzy rule based

	Temperature	Humidity	Speed
Universe of discourse	$X = [0,50]$	$Y = [0,20]$	$S = [0,10]$
Observed Value	$x = \{TLow, TMedium, THigh\}$	$y = \{HLow, HMedium, HHigh\}$	$s = \{SLow, SMedium, SHigh\}$
Membership function	$\widetilde{TLow} = tfn[-10,0,10]$	$\widetilde{HLow} = trfn[-10,-5,5,10]$	$\widetilde{SLow} = trfn[-2,-1,1,3]$
	$\widetilde{TMedium} = trfn[5,20,30,45]$	$\widetilde{HMedium} = tfn[6,10,14]$	$\widetilde{SMedium} = trfn[2,4,6,8]$
	$\widetilde{THigh} = tfn[40,50,60]$	$\widetilde{HHigh} = trfn[10,15,25,30]$	$\widetilde{SHigh} = trfn[7,9,11,15]$

Fuzzy Rule

Rule 1	If (Temperature is T_Low) or (Humidity is HLow) then (Speed is SHigh)
Rule 2	If (Temperature is TMedium) and (Humidity is HLow) then (Speed is SMedium)
Rule 3	If (Temperature is THigh) and (Humidity is HLow) then (Speed is SLow)
Rule 4	If (Temperature is TMedium) or (Humidity is HMedium) then (Speed is SMedium)
Rule 5	If (Temperature is T_Low) and (Humidity is HMedium) then (Speed is SMedium)
Rule 6	If (Temperature is THigh) and (Humidity is HMedium) then (Speed is SLow)
Rule 7	If (Temperature is THigh) or (Humidity is HHigh) then (Speed is SLow)
Rule 8	If (Temperature is TMedium) and (Humidity is HHigh) then (Speed is SLow)
Rule 9	If (Temperature is T_Low) and (Humidity is HHigh) then (Speed is SLow)

Determine the value of speed when temperature and humidity are 7 and 16 respectively. Use $< min >$ for And, $< max >$ for Or method, $< min >$ for implication, $< max >$ for aggregation and $< mean\ of\ maxima >$ for defuzzification. **20**

2. We define the dissimilitude relation for fuzzy relation $\tilde{R} \subseteq \tilde{A} \times \tilde{A}$ if it satisfies antireflexive, symmetric and min-max transitive relation $\left(\mu_R(x,z) \leq \wedge_y\big(\mu_R(x,y) \vee \mu_R(y,z)\big)\right.$. Consider the relation $\tilde{R}$ on $A \times A$ where $A = \{a, b, c, d\}$

$\tilde{R}$	a	b	c	d
a	0.0	0.2	0.1	0.0
b	0.2	0.0	0.1	0.2
c	0.1	0.1	0.0	0.3
d	0.0	0.2	0.3	0.0

Check, whether $\tilde{R}$ is dissimilitude relation or not. Justify. **10**

3. Determine the relation matrix for the implication rule 'IF $< x\ is\ \tilde{A} >$ THEN $< y\ is\ \tilde{B} >$' using Yager class with $w = 2$ for $\tilde{A} = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.9), (x_4, 0.1)\}, \tilde{B} = \{(y_1, 0.1), (y_2, 0.9), (y_3, 0.1), (y_4, 0.9)\}$. **15**

4. For the single server Markovian queue, the expectations are given by $\rho = \frac{\lambda}{\eta}, L = \frac{\rho}{1-\rho}, L = L_q + \rho, W = \frac{L}{\lambda}, W = W_q + \frac{1}{\eta}$ where λ and η are mean arrival and service rate respectively. If λ is approximated by $\tilde{\lambda} = trfn[1,2,4,7]$ and η is approximated by $\tilde{\eta} = tfn[8\ 12\ 14]$. Determine the membership grade function of approximated $\widetilde{W_q}$ using $\alpha -$cut approach and extension principle. **15**

5. For the two-unit parallel machining system, the reliability of the system R_s is expressed as $\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2}$ and for two-unit series machining system, the reliability of the system R_s is expressed as $R_s = R_1 R_2$. The membership and non-membership grade function for approximated reliability of unit 1 is given by $trfn(2,3,4,5)$ and $trfn(1,3,4,6)$ respectively. Similarly, The membership and non-membership grade function for approximated reliability of unit 2 is given by $tfn(5,6,8)$ and $tfn(4,6,10)$ respectively. Determine the approximate membership and non-membership grade function of the reliability of the following parallel-series system.

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6. Let the universe of discourse $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ respectively. Assume that the proposition IF $< x\ is\ \tilde{A} >$ THEN $< y\ is\ \tilde{B} >$ ELSE $< y\ is\ \tilde{C} >$ where $\tilde{A} = \{(x_1, 0.5), (x_2, 1), (x_3, 0.6)\}$, $\tilde{B} = \{(y_1, 1), (y_2, 0.4)\}$, and $\tilde{C} = \{(y_1, 0.8), (y_2, 0.2)\}$. Assume that the fact expressed by $< y\ is\ \tilde{B}' >$ is given where $\tilde{B}' = \{(y_1, 0.9), (y_2, 0.7)\}$. Deduce $< x\ is\ \tilde{A}' >$ Zadeh's implication max-min principle. **15**