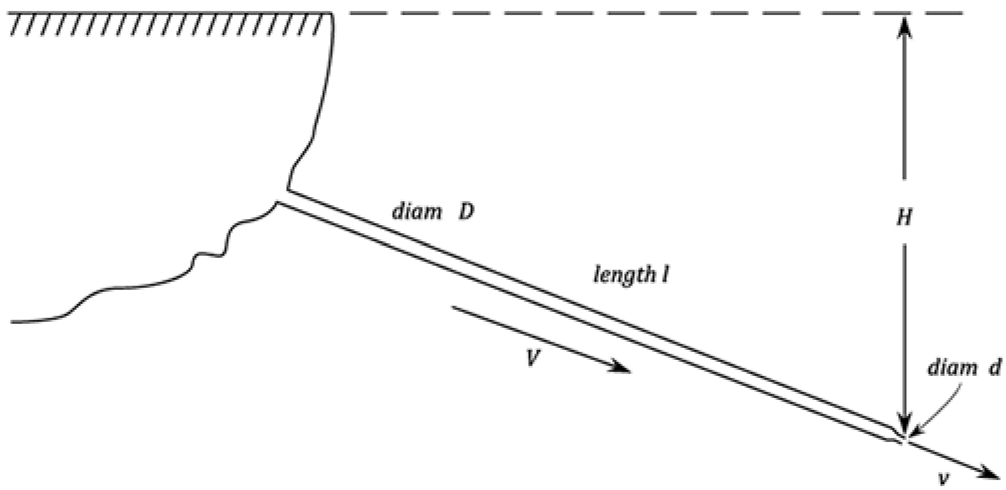
Q1. Prove that the maximum power transmission through nozzle is $\frac{d}{D} = \sqrt[4]{\frac{D}{8fl}}$ by applying Bernoulli's theorem in the given diagram. Neglect the minor losses.



Applying Bernoulli's equation across the nozzle and neglecting losses:

$$H = \frac{4flV^{2}}{2gD} + \frac{v^2}{2g}$$

And using the continuity equation:

$$VD^2 = vd^2$$

$$H = \frac{4fl}{D} \times \left(\frac{d}{D}\right)^4 \times \frac{v^2}{2g} + \frac{v^2}{2g} = \frac{v^2}{2g}\left[\frac{4fl}{D}\left(\frac{d}{D}\right)^4 + 1\right]$$

$$\therefore \quad v = \sqrt{\frac{2gH}{\frac{4fl}{D} \times \left(\frac{d}{D}\right)^4} + 1}$$

The power output of the nozzle is given by:

$$P = \frac{\omega\ a\ v^3}{2g \times 550}$$

$$= \frac{\omega\ \pi d^2}{4 \times 2g \times 550} \left(\frac{2gH}{\frac{4fl}{D} \times \left(\frac{d}{D}\right)^4} + 1 \right)^{\frac{3}{2}}$$

$$= Kd^2 \left(\frac{4fl}{D} \times \left(\frac{d}{D}\right)^4 + 1 \right)^{-\frac{3}{2}}$$

The nozzle diameter for the maximum power transmission will occur when $\frac{dP}{d(d)} = 0$

i.e.

$$2d \left[\frac{4fl}{D} \times \left(\frac{d}{D}\right)^4 + 1 \right]^{-\frac{3}{2}} = \frac{3}{2} d^2 \left[4fl \left(\frac{d}{D}\right)^4 + 1 \right]^{-\frac{5}{2}} \times 4 \times \frac{4fl}{D} \times \frac{d^3}{D^4}$$

Multiply both sides by

$$\left[\frac{4fl}{D} \times \left(\frac{d}{D}\right)^4 + 1 \right]^{\frac{5}{2}}$$

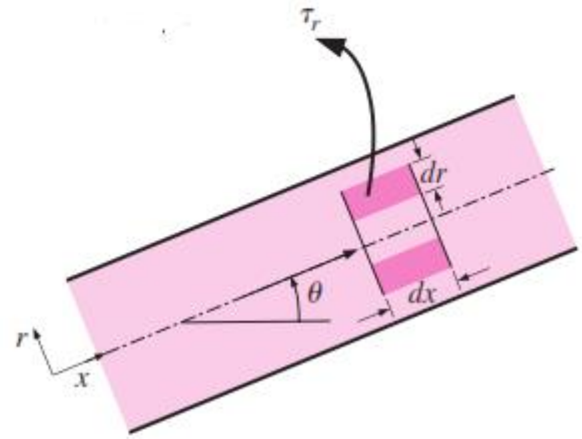
$$\therefore \quad \frac{4fl}{D} \times \left(\frac{d}{D}\right)^4 + 1 = \frac{3}{2} \times \frac{d \times 2 \times 4fld^3}{D^5} = \frac{12 \times fl}{D} \left(\frac{d}{D}\right)^4$$

Or:

$$\frac{8fl}{D} \left(\frac{d}{D}\right)^4 = 1$$

$$\therefore \quad \frac{d}{D} = \sqrt[4]{\frac{D}{8fl}}$$

Q2. Consider a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with an inclined pipe in fully developed laminar flow, as shown in Fig



Derive the expression for the *average velocity* and the *volume flow rate* relations for laminar flow through inclined pipe.

Relations for inclined pipes can be obtained in a similar manner from a force balance in the direction of flow. The only additional force in this case is the component of the fluid weight in the flow direction, whose magnitude is

$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx)\sin\theta$$

Where, θ is the angle between the horizontal and the flow direction. The force balance now becomes

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0$$

Which results in the differential equation

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

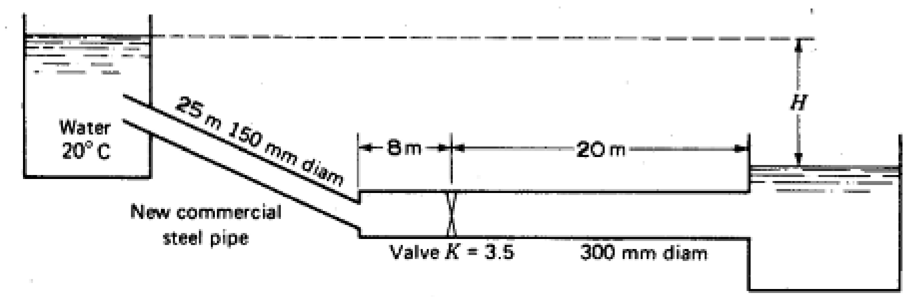
The velocity profile can be shown to be:

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

It can also be shown that the *average velocity* and the *volume flow rate* relations for laminar flow through inclined pipes are, respectively,

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L} \quad \text{and} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

Q 3. Sketch the HGL and EGL for Fig.(). The friction factor is f = 0.015 for all pipes.



$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(V^2/2g)$$

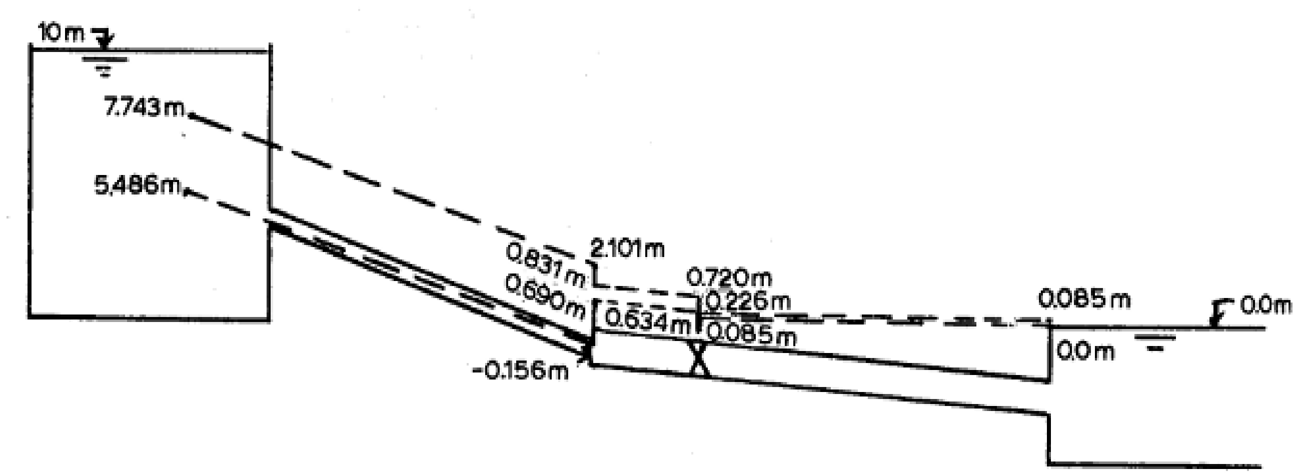
$$h_f = (0.015)[25/0.150]\{V_1^2/[(2)(9.807)]\} + (0.015)[(20+8)/0.300]\{(V_1/4)^2/[(2)(9.807)]\} = 0.1319V_1^2$$

$$h_m = KV^2/2g = [1 + (1 - \tfrac{1}{4})^2 + 3.5/16 + \tfrac{1}{16}]\{V_1^2/[(2)(9.807)]\} = 0.09400V_1^2$$

$$h_L = 0.1319V_1^2 + 0.09400V_1^2 = 0.2259V_1^2 \qquad 0 + 0 + 10 = 0 + 0 + 0 + 0.2259V_1^2 \qquad V_1 = 6.653 \text{ m/s}$$

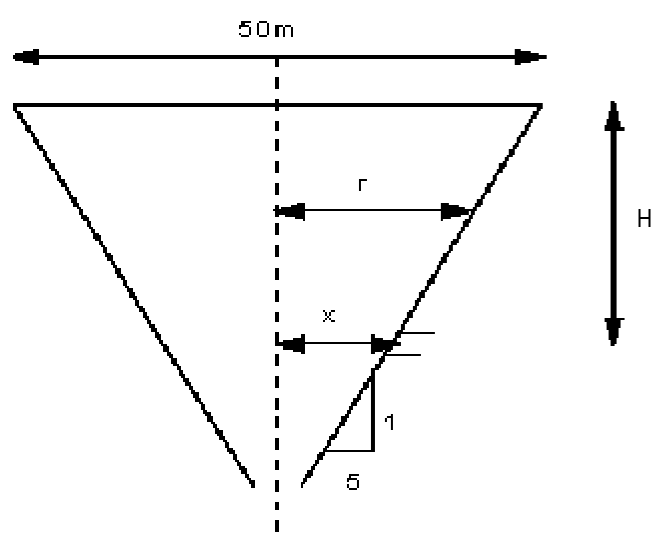
$$V_1^2/2g = 6.653^2/[(2)(9.807)] = 2.257 \text{ m} \qquad V_2 = 6.653/4 = 1.663 \text{ m/s} \qquad V_2^2/2g = 1.663^2/[(2)(9.807)] = 0.141 \text{ m}$$

Friction losses are $(f_1)(L/D)(v_1^2/2g) = 5.642$ m, $(f_2)(L/D)(v_2^2/2g) = 0.197$ m. Minor losses are $(V_1 - V_2)^2/2g =$ 1.270 m, $3.5V_2^2/2g = 0.494$ m.



Q 4. A reservoir is circular in plan and the sides slope at an angle of $\tan^{-1}(1/5)$ to the horizontal. When the reservoir is full the diameter of the water surface is 50m. Discharge from the reservoir takes place through a pipe of diameter 0.65m, the outlet being 4m below top water level.

Determine the time for the water level to fall 2m assuming the discharge to be $0.75a\sqrt{2gH}$ cumecs where *a* is the cross sectional area of the pipe in m^2 and *H* is the head of water above the outlet in m.



From the question: H = 4m

$a = \pi(0.65/2)^2 = 0.33m^2$

$$Q = 0.75a\sqrt{2gh}$$
$$= 1.0963\sqrt{h}$$

In time Δt the level in the reservoir falls Δh, so

$$Q\,\delta t = -A\,\delta h$$
$$\delta t = -\frac{A}{Q}\,\delta h$$

Integrating give the total time for levels to fall from h_1 to h_2.

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$

As the surface area changes with height, we must express *A* in terms of *h.*

$A = \pi r^2$

But *r* varies with *h*.

It varies linearly from the surface at $H = 4m$, $r = 25m$, at a gradient of $\tan^{-1} = 1/5$.

$r = x + 5h$

$25 = x + 5(4)$

$x = 5$

$so\ A = \pi(5 + 5h)^2 = (25\pi + 25\pi h^2 + 50\pi h)$

Substituting in the integral equation gives

$$T = -\int_{h_1}^{h_2} \frac{25\pi + 25\pi h^2 + 50\pi h}{1.0963\sqrt{h}}\,dh$$
$$= -\frac{25\pi}{1.0963}\int_{h_1}^{h_2} \frac{1 + h^2 + 2h}{\sqrt{h}}\,dh$$
$$= -71.641\int_{h_1}^{h_2} \frac{1}{\sqrt{h}} + \frac{h^2}{\sqrt{h}} + \frac{2h}{\sqrt{h}}\ dh$$
$$= -71.641\int_{h_1}^{h_2} h^{-1/2} + h^{3/2} + 2h^{1/2}\ dh$$
$$= -71.641\left[2h^{1/2} + \frac{2}{5}h^{53/2} + \frac{4}{3}h^{3/2}\right]_{h_1}^{h_2}$$

From the question, $h_1 = 4m$ $h_2 = 2m$*, so*

$$T = -71.641\left[\left(2\times 4^{1/2} + \frac{2}{5}\times 4^{53/2} + \frac{4}{3}\times 4^{3/2}\right) - \left(2\times 2^{1/2} + \frac{2}{5}\times 2^{53/2} + \frac{4}{3}\times 2^{3/2}\right)\right]$$
$$= -71.641\left[(4 + 12.8 + 10.667) - (2.828 + 2.263 + 3.77)\right]$$
$$= -71.641\left[27.467 - 8.862\right]$$
$$= 1333 \text{ sec}$$

Q 5. For a network of pipelines, such as that described in part (a), show that the flow correction term in an iterative head balance calculation is given by

$$\Delta Q = \frac{-\sum h_f}{2\sum (h_f/Q)}$$

Solution:

Starting with $h_f = K\,Q^n$ Normally $n = 2$ so $h_f = K\,Q^2$

Differentiate to get $dh_f = 2KQdQ = \dfrac{2KQ^2dQ}{Q}$ and since $K\,Q^2 = h_f$

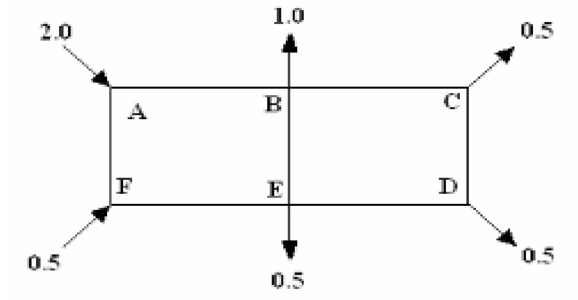
$$dh_f = \frac{2h_f dQ}{Q} \text{ or } dQ = \frac{Qdh_f}{2h_f}$$

If this relationship holds approximately true for finite changes then $\delta h_f = \dfrac{2h_f\delta Q}{Q}$ or $\delta Q = \dfrac{Q\,\delta h_f}{2h_f}$

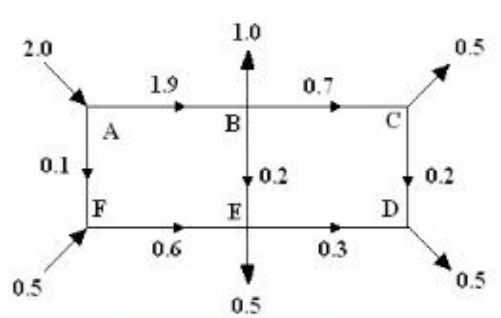
In a balance of heads, the flow is corrected until $\Delta\theta = 0$ so the correction factor to be used for each pipe is $\delta Q = \dfrac{-Q\,\delta h_f}{2h_f} = \dfrac{-\delta h_f}{2\left(\dfrac{h_f}{Q}\right)}$ (The correction must be to reduce the flow rate).

For a network we must total all the terms to give the total correction factor of $\Delta Q = \dfrac{-\sum h_f}{2\sum (h_f/Q)}$

Q 6. The diagram shows two loops of a horizontal network with inflows and outflows in m^3 /s. The K values of the seven pipes are given in the table. The pressure head at node A is 25m. Calculate the flow rate through each pipe and the pressure head at each node. No more than two rounds of iteration are required, and final values of pressure heads may be rounded to the nearest meter.



Pipe	AB	BC	CD	DE	BE	EF	AF
K(s^2/m^5)	2	2	20	20	10	10	10



Data shown for initial guess

Start with loop ABEFA

PIPE	K	Q	h_f	h_f/Q
AB	2	1.9	7.22	3.8
BE	10	0.2	0.4	2
EF	10	-0.6	-3.6	6
FA	10	-0.1	-0.1	1
		Totals	3.92	12.8

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{3.92}{2 \text{ x } 12.8} = -0.153$$

Correct all flows in this loop by adding -0.153

Now do loop BCDEB

PIPE	K	Q	h_f	h_f/Q
BC	2	0.7	0.98	1.4
CD	20	0.2	0.8	4
DE	20	-0.3	-1.8	6
BE	10	0.04688	0.021973	0.46875
		Totals	0.001973	11.86875

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.002}{2 \text{ x } 11.869} = -0.000083$$

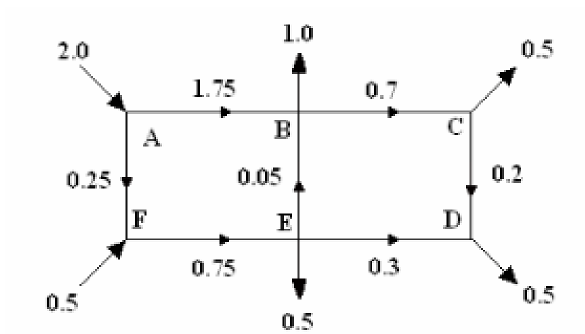
Correct loop 2 The initial guess was so good that the correction is minor

Second iteration of loop 1 is:

PIPE	K	Q	h_f	h_f/Q
AB	2	1.74688	6.10314	3.49375
BE	10	-0.0468	-0.02189	0.467919
EF	10	-0.7531	-5.67197	7.53125
FA	10	-0.2531	-0.64072	2.53125
			-0.23145	14.02417

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.231}{2 \text{ x } 14.02} = -0.00825$$

Final solution is



Pipe	AB	BC	CD	DE	BE	EF	AF
Q	1.75	0.7	0.2	-0.3	±0.05	-0.75	-0.25

Head at A is 25 m and rounding off the h_f values
Head B is 25 - 6 = 19 m
Head at C is 19 – 1 = 18 m
Head at D = 18 - 1 = 17 m
Head at E = 17 + 2 = 19 m or 19 - 0 = 19 m
Head at F = 19 + 6 = 25 m or 25 -1 = 24 m error due to rounding off values.

1. The velocity profile in a laminar boundary layer on a flat plate is to be modelled by the cubic expression $u/u_1 = a_0 + a_1y + a_2y^2 + a_3y^3$, where u is the velocity a distance y from the wall and u_1 is the main stream velocity. Explain why a_0 and a_2 are zero and evaluate the constants a_1 and a_3 in terms of the boundary layer thickness δ. Define the momentum thickness θ and show that it equals 39 δ / 280.

Solution:

At $y = 0$, $u = 0$ so it follows that $a_o = 0$

$d^2u/dy^2 = 0$ @ $y = 0$ so $a_2 = 0$. Show for yourself that this is so.

The law is reduced to $u/u_1 = a_1y + a_3y^3$

At $y = \delta$, $u = u_1$ so $1 = a_1\delta + 3a_3\,\delta^2$

hence $a_1 = (1 - a_3\delta^3)/\delta$

Now differentiate and $du/dy = u_1\,(a_1 + 3a_3y^2)$

at $y = \delta$, du/dy is zero so $0 = a_1 + 3a_3\,\delta^2$ so $a_1 = -3a_3\delta^2$

Hence by equating $a_1 = 3/2\delta$ and $a_3 = -1/2\delta^3$

Now we can write the velocity distribution as $u/u_1 = 3y/2\delta - (y/\delta)^3/2$

and $du/dy = u_1\,\{3/2\delta + 3y^2/2\delta^3\}$

If we let $y/\delta = \eta$ $u/u_1 = \{3\eta/2 + (\eta)^3/2\}$

The momentum thickness is

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right] dy \quad \text{but } dy = \delta d\eta$$

$$\theta = \int_0^1 \left[\frac{3\eta}{2} - \frac{\eta^3}{2}\right]\left[1 - \frac{3\eta}{2} - \frac{\eta^3}{2}\right] d\eta$$

Integrating gives: $$\theta = \delta\left[\frac{3\eta^2}{4} - \frac{\eta^4}{8} - \frac{9\eta^3}{12} - \frac{\eta^7}{28} + \frac{3\eta^5}{10}\right]$$

Between the limits $\eta = 0$ and $\eta = 1$ this evaluates to $\theta = 39\delta/280$

2. The velocity profile in a laminar boundary layer is sometimes expressed in the formula:

$$\frac{u}{u_1} = a_o + a_1\frac{y}{\delta} + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 + a_4\left(\frac{y}{\delta}\right)^4$$

Where u_1 is the velocity outside the boundary layer and δ is the boundary layer thickness. Evaluate the coefficients a_0 to a_4 for the case when the pressure gradient along the surface is zero.

Solution:

Part A

$$\frac{u}{u_1} = a_0 + a_1\frac{y}{\delta} + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 + a_4\left(\frac{y}{\delta}\right)^4$$

Boundary conditions

Where y = 0, u = 0 hence $a_0 = 0$

Where $y = \delta$, $u = u_1$ $\quad 1 = a_1 + a_2 + a_3 + a_4$ ....(A)

Differentiate with respect to y

$$\frac{1}{u_1}\frac{du}{dy} = \frac{a_1}{\delta} + 2a_2\left(\frac{y}{\delta^2}\right) + 3a_3\left(\frac{y^2}{\delta^3}\right) + 4a_4\left(\frac{y^3}{\delta^4}\right)$$

Where $y = \delta$, du/dy = 0

$$0 = \frac{a_1}{\delta} + 2a_2\left(\frac{1}{\delta}\right) + 3a_3\left(\frac{1}{\delta}\right) + 4a_4\left(\frac{1}{\delta}\right)$$

$0 = a_1 + 2a_2 + 3a_3 + 4a_4$ ....(B)

Differentiate a second time.

$$\frac{1}{u_1}\frac{d^2u}{dy^2} = 2a_2\left(\frac{1}{\delta^2}\right) + 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right)$$

Where y = 0 , $d^2u/dy^2 = 0$ hence $0 = 2a_2\left(\frac{1}{\delta^2}\right)$ Hence $a_2 = 0$

(A) becomes $\quad 1 = a_1 + a_3 + a_4$

(B) becomes $\quad 0 = a_1 + 3a_3 + 4a_4$

Subtract $\quad 1 = 0 - 2a_3 - 3a_4$ ....(C)

The second differential becomes

$$\frac{1}{u_1}\frac{d^2u}{dy^2} = 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right)$$

Where $y = \delta$, $d^2u/dy^2 = 0$

$$0 = 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right) 6a_3\left(\frac{1}{\delta^2}\right) + 12a_4\left(\frac{1}{\delta^2}\right) = 6a_3 + 12a_4 \text{ ....(D)}$$

Divide through by 3 $\quad 0 = 2a_3 + 4a_4$ ....(E)

Add (C) and (E) $\quad a_4 = 1$

Substitute into (E) $\quad 0 = 2a_3 + 4 \quad a_3 = -2$

Substitute into (A) $\quad 1 = a_1 - 2 + 1 \quad a_1 = 2$

Hence

$$\frac{u}{u_1} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + 2\left(\frac{y}{\delta}\right)^4$$

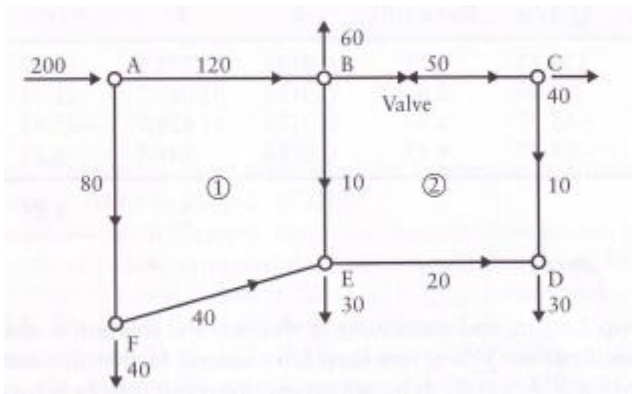
3. In the network shown in figure, a valve in BC is partially closed to produce a local head loss of $\frac{10V_{BC}^2}{2g}$.

Analyse the flows in the network.

Pipe	AB	BC	CD	DE	BE	EF	AF
Length (m)	500	400	200	400	200	600	300

Diameter (mm)	250	150	100	150	150	200	250

Note: Roughness of all pipes is 0.06 mm.



Solution:

The procedure is identical with that of the previous problem. K_{BC} is now composed of the valve loss coefficient and the friction loss coefficient.

With the initial assumed flows shown in the table below, $Q_{BC} = 50$ L/s; $Re = 3.7 \times 10^5$; $k/D = 0.0004$; $\lambda = 0.0174$ (from the Moody chart). Hence, $K_f = 7573$, $K_m = 1632$ and $K_{BC} = 9205$.

	Pipe	k/D	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{m}{m^3/s}\right)$
Loop 1	AB	0.00024	120.00	5.41	0.0157	664.2	9.56	79.70
	BE	0.00040	10.00	0.75	0.0208	4 526.5	0.45	45.26
	EF	0.00030	−40.00	2.25	0.0175	2 711.2	−4.34	108.45
	FA	0.00024	−80.00	3.61	0.0163	1413.7	−2.65	33.10
						$\sum$	3.03	266.51

$\Rightarrow \Delta Q = -5.69$ L/s.

	Pipe	k/D	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{m}{m^3/s}\right)$
Loop 2	BC	0.0004	50.00	3.75	0.0174	9 205.2	23.01	460.26
	CD	0.0006	10.00	1.13	0.0205	33 877.0	3.39	338.77
	DE	0.0004	−20.00	1.50	0.0190	8 226.0	−3.29	164.52
	EB	0.0004	−4.31	0.32	0.0242	5 266.4	−0.10	22.70
						$\sum$	23.01	986.25

$\Rightarrow \Delta Q = -11.67$ L/s.

Proceeding in this way the solution is obtained within a small limit on $\sum h$ in any loop:

Final values

Pipe	AB	BE	FE	FA	BC	CD	ED
Q (L/s)	111.52	16.48	48.48	88.48	35.05	4.95	34.95
h_L (m)	8.31	1.15	6.26	3.20	11.57	0.91	9.52

EXAM QUESTIONS

Q1. Two reservoirs with 15 m difference in their water levels are connected by a 300 mm diameter pipeline of 3000 m length. Calculate the discharge. If a parallel pipe line of 300 mm diameter is attached to the last 1500 m length of existing pipe, determine the modified discharge. Take only wall friction into account. Assume f = 0.04 in Darcy-Weisbach formula.

Solution:

$$H_L = \frac{f\,L\,V^2}{2gd} = \frac{f\,L\left(\frac{Q}{\left(\frac{\pi}{4}\right)d^2}\right)^2}{2gd}$$

$$H_L = \frac{8}{g\pi^2}\frac{f\,L\,Q^2}{d^5}$$, H_L = 15 m

In the 1st case, let Q1 be the discharge flowing through the pipe of 0.3 m dia and 3000 m length, having f = 0.04. Then

$$15 = \frac{8}{g\pi^2}\frac{0.04*3000*Q1^2}{0.3^5}$$

Or Q1 = 0.06063 cu.m/s

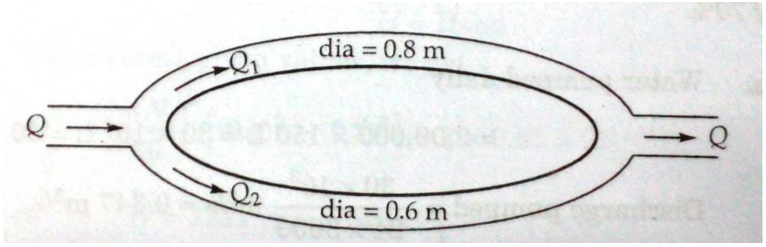
In the 2nd case, when additional pipe of 1500 m length is laid in the second half in parallel to the original pipe, let the discharge through the initial length of pipe be Q2. The discharge through the original 1500 m length will thus be Q2, and that through the two parallel pipes shall be Q2/2 each. The total head loss will then be,

$$15 = \frac{8}{g\pi^2}\frac{0.04}{0.3^5}\left[1500Q2^2 + 1500\left(\frac{Q2}{2}\right)^2\right]$$

Q2 = 0.0767 cu.m/s = Modified discharge

Q2. Two pipes of lengths 2500 m each and diameters of 80 cm and 60 cm respectively are connected in parallel. The friction factor for each pipe is 4f = 0.024. Total flow is equal to 250 litres per second. Find the discharge in each pipe.

Solution:



Discharge in a pipe is given by Darcy's- Weisbach equation as, $H_L = \frac{4f\,L\,V^2}{2gd}$

Where, $V^2 = Q^2/A^2$

$$H_L = \frac{4fL}{2gd\pi^2}\frac{16\,Q^2}{d^4} = \frac{8}{g\pi^2}\frac{4f\,L\,Q^2}{d^5}$$

For 1st pipe,

$$H_L = \frac{8}{9.81 * \pi^2}\frac{0.024 * 2500 * \;Q1^2}{0.8^5}$$

$H_L = \;15.129\;Q1^2$ -----------(i)

For 2nd pipe,

$$H_L = \frac{8}{9.81 * \pi^2}\frac{0.024 * 2500 * \;Q2^2}{0.6^5}$$

$H_L = \;63.76\;Q2^2$ --------------(ii)

Equating (i) and (ii),

$15.129\ Q1^2 = 63.76\ Q2^2$

Q1 = 2.05 Q2

But, Q1 + Q2 = Q = 250 litres/s

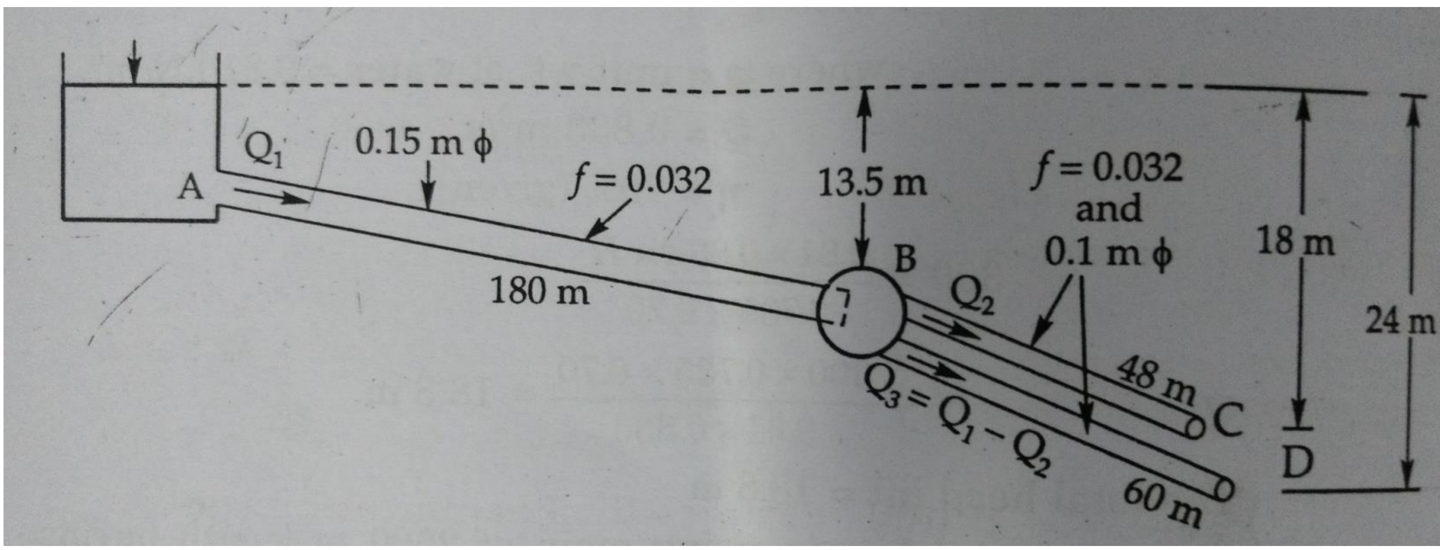
2.05 Q2 + Q2 = 250

Q2 = 82 litres/s

Q1 = 168 litres/s

Water flows from a reservoir through a pipe of 0.15m dia and 180m long to a point 13.5m below the open surface of the reservoir. Here it branches into two pipes, each of 0.1m dia, one of which is 48m long discharging to atmosphere at a point 18m below reservoir level, and the other 60m long discharging to atmosphere 24m below reservoir level. Assuming constant friction coefficient of 0.032, Calculate the discharge from each pipe. Neglect any losses at the junction.

Ans.]



H_L between ABC = 18m

Using Eq. $H_L = 8f' LQ^2 /(g \pi^2 d^5)$

$(8*0.032/(9.81*3.14*3.14))[(180Q_1^2/(0.15^5)) + (48Q_2^2/(0.1^5))] = 18$

$Q_2 = (0.0014168 - 0.4938Q_1^2)^{0.5}$

And H_L between ABD = 24m

$(8*0.032/(9.81*3.14*3.14))[(180Q_1^2/(0.15^5)) + (60(Q_1 - Q_2)^2/(0.1^5))] = 24$

$Q_1^2 + 2.5312[Q_1 - (0.0014168 - 0.4938Q_1^2)^{0.5}]^2 = 0.003825$

Solving by hit and trial, we get

Q_1 = 0.0458 cumecs

Q_2 = 0.0195 cumecs

$Q_3 = Q_1 - Q_2$ = 0.0263 cumecs

Q.2]

For water supply of a town, water is pumped from a river 3km away into a reservoir. The maximum difference of levels of water in a river and the reservoir is 20m. The population of the town is 50,000 and per capita water demand is 120 l/day. If the pumps are to operate for atotal of 8 hrs and the efficiency of pumps is 80%, find the H.P. of the pumps. Assume friction factor as 0.03, the velocity of flow as 2m/s and maximum daily demand as 1.5 times the average daily demand.

Ans.]

Max daily demand = 1.5*50000*120 l/day = $9*10^6$ l/day = $9*10^3\ m^3$ /day

This much amount of water to be lifted by pumps in 8 hrs.

Discharge to be lifted by the pumps = $9*10^3/(8*60*60)$ = 0.3125 m^3 /s

Velocity of flow v = 2m/s (given)

Area of pipe = Q/v = 0.3125/2 = 0.15625 m^2

Dia of pipe = $(4*0.15625/\pi)^{0.5}$ = 0.446m

Head loss through the pipe $H_{L1} = f'Lv^2/2gd = 0.03*3000*2^2/(2*9.81*0.446)$ = 41.14m

H_{L1} = 41.14m

H_{L2} = 20m (given)

Total head loss H_L = 41.14 + 20 = 60.14m

Required H.P. of pumps = $Y_w Q\ H_L/(0.735\eta)$ = 9.81*0.3125*61.14/(0.735*0.8) = 318.76 H.P.

Laminar flow problem:

Calculate the diameter of a vertical pipe needed for flow of a liquid at a Reynolds number of 1200 when the pressure remains constant throughout the pipe, Kinematic viscosity of the fluid = 1.92 X 10^{-3} sq.m/s

Solution:

Reynolds number, Re = VD/v = 1200

VD = 1200 * 1.92 X 10^{-3} = 2.304 ------------------------------(i)

Energy gradient , Se = $\frac{\left\{\left(\frac{p1}{\gamma}+Z1\right)-\left(\frac{p2}{\gamma}+Z2\right)\right\}}{L}$

Where, p1 = p2

Also for a vertical pipe (Z1-Z2) = L

Se = (Z1-Z2)/(Z1-Z2) = 1.0

For laminar flow

$S_L = h_L/L = (32\mu V)/(\gamma D^2) = (32vV)/(gD^2)$

In the present case, $(32vV)/(gD^2) = 1$

V/D^2 = g/(32v) = (9.81)/(32*1.92 X 10^{-3}) = 159.67 --------------------------(ii)

From (i) & (ii), $\frac{V/D^2}{VD}$ = $1/D^3$ = 159.67/2.304 = 69.3

D = 0.243 m

OBJECTIVE QUESTIONS:

Q1. In a steady flow of oil in the fully developed laminar regime, the shear stress is:

(a) Constant across the pipe

(b) Maximum at the centre an decreases parabolically towards the pipe wall boundary

(c) Zero at the boundary and increases linearly towards the centre.

(d) Zero at the centre and increases towards the pipe wall.

Ans. (d)

Q2. For flow through a horizontal pipe, the pressure gradient dp/dx in the flow direction is:

(a) +ve (b) 1 (c) Zero (d) –ve

Ans. (d) For flow through a horizontal pipe, the pressure gradient dp/dx in the flow direction is –ve.

Q3. The frictional head loss through a straight pipe (h_f) can be expressed as

$h_f = \frac{flv^2}{2gD}$ for both lami nar and turbulent flows. For a laminar flow, 'f' is giv en by (Re is the Reynolds Number based on pipe d iameter)

(a) 24/Re (b) 32/Re (c) 64/Re (d) 128/Re

Ans. (c) Here 'f' is friction factor.

Multiple choice questions

1. An oil of kinematic viscosity 0.25 stokes flows through a pipe of diameter 10 cm. The flow is critical at a velocity of

a. 7.2 m/s b. 5.0 m/s c. 0.5 m/s d. 0.72 m/s

2. In a circular pipe of certain length carrying oil at a Reynolds number 100, it is proposed to triple the discharge. If the viscosity remains unchanged, the power input will have to be

a. decreased to 1/3 its original value

b. increased by 100 %

c. increased to 3 times the original value

d. increased to 9 times its original value

3. In a laminar flow through a circular pipe of diameter 20 cm, the maximum velocity is found to be 1 m/s. The velocity at the radial distance of 5 cm from the axis of the pipe will be

a. 0.25 m/s b. 0.50 m/s c. 0.75 m/s d. 0.10 m/s

4. In laminar flow between two fixed parallel plates, the shear stress is

a. Constant across the passage

b. maximum at the center and zero at the end

c. zero all through the passage

d. maximum at the boundary and zero at the centre

5. The creeping motion obeys the Stokes law up to a critical Reynolds number of value

a. 0.001 b. 1.0 c. 100 d. 2000

6. Two pipe systems in series are said to be equivalent when

a. the average diameter in both systems is the same

b. the average friction factor in both systems is same

c. the total length of the pipe is the same in both the systems

d. the discharge under the same head is the same in both systems

7. The minor loss due to sudden contraction is due to

a. flow contraction

b. expansion of flow after sudden contraction

c. boundary friction

d. cavitation

8. Two identical pipes of length L, diameter D and friction factor f, are connected in series between two reservoirs. The size of a pipe of length L and of the same friction factor f, equivalent to the above pipeline is

a. 0.5 D b. 0.87 D c. 1.15 D d. 1.40 D

9. The maximum transmission of power through a pipeline with a total head H, the head loss due to friction h_f is given by h_f =

a. H/3 b. (2/3) H c. H/2 d. 0.1 H

10. Hydraulic grade line for flow in a pipe of constant diameter is

a. always above the centerline of the pipe

b. always above the energy grade line

c. always sloping downwards in the direction of the flow

d. coincides with the pipe centreline