BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2022-23
CE F320: Design of Reinforced Concrete Structures
Mid-semester exam; 3/Nov./22; Open book (IS 456:2000, Textbook, and Handwritten Class Notes)

Duration: 90 minutes **MM. :80**

Design Philosophy: Limit state method as per IS 456:2000

Instructions:

- In the numerical-type questions, full marks will **NOT** be given if detailed calculation steps are not presented, as required.
- All given loads are to be considered as unfactored/service/working loads.

Design aid:

Values of f_{sc} in MPa(N/mm^2) for Fe 415 steel and d'/d ratio are given in the table below (symbols have their general meaning).

f_{sc}				
Grade of steel	d'/d			
	0.05	0.10	0.15	0.20
Fe 415	**355.1**	**351.9**	**342.4**	**329.2**

Q.1 Fill in the blanks **[MM. 1 x 5 = 5]**

(a) Load value that has a 95 percent probability of not being exceeded during the life of the structure is called ____________________.

(b) Four factors responsible for the difference in the compressive strength of concrete in an actual structure from the strength obtained in a standard uniaxial compression test are ______________, ______________, ____________, and __________.

(c) A state of impending failure, beyond which a structure ceases to perform its intended function satisfactorily, in terms of either safety or serviceability, i.e., it either collapses or becomes unserviceable, is called as ________________.

(d) The maximum compressive strain in concrete under flexure; axial; combined axial and flexure is equal to ________________; ____________________; _____________________, respectively.

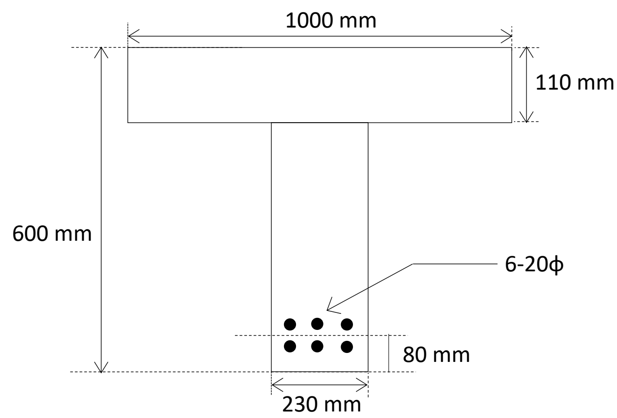
(e) The expression for calculating $x_{u,max}$ in terms of steel grade, steel modulus, and effective depth is ______________.

Q.2 Design a rectangular RCC beam located in a coastal area but sheltered from saturated salt air. The beam is simply supported over 250 mm thick masonry walls that are 6 m apart center to center. In addition to self-weight, the beam has to carry a sustained dead load of 5 kN/m and a live load of 10 kN/m. The width of the beam is equal to the thickness of the walls. The grade of steel rebars is Fe 415. The effective cover to reinforcement can be taken as 50 mm for trial calculations, wherever required. Assume the maximum size of coarse aggregates as 20 mm, and the diameter of stirrups as 8 mm. Also, sketch the reinforcement details provided at the most critical section. **[MM. 20]**

Q.3 Redesign the beam of Q.2 and sketch the reinforcement details provided at the most critical section if, due to architectural considerations, the size of the beam is fixed as 250 mm x 400 mm, and the beam is required to carry an additional concentrated sustained dead load of 30 kN at the mid-span over and above the loads given in Q.2.

[MM. 30]

Q.4 Determine the ultimate moment carrying capacity of a simply supported isolated T-beam having an effective span of 6 m and a cross-section, as shown in the Figure below. The grade of steel rebars is Fe 415, and the exposure condition is mild. **[MM. 25]**



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SOLUTIONS

A.1:Sol:

a) Characteristic load.

b) Duration of loading; Size of member; Strain gradient; Multi-axial state of stress.

c) Limit state

d) Flexure: 0.0035**;** Axial: 0.002**;** Combined axial and flexure 0.0035 - 0.75 times the strain at least compressed extreme fiber.

e) $x_{u,max} = \frac{0.0035 \times d}{0.0055 + \frac{0.87 f_y}{E_S}}$

A.2: Sol:

$f_{ck} = 25N/mm^2$(Concrete surfaces sheltered from saturated salt air in coastal area: Moderate exposure (Table 3 IS 456:2000), min. grade of concrete for moderate exposure: M 25 (Table 5 IS 456:2000)); $f_y = 415\ N/mm^2$; $b = 250\ mm$; c/c distance between supports = 6 m

Step#1 Calculation of the trial effective depth, and trial effective span of the beam. **[3]**

As per [23.2.1, IS 456:2000] for simply supported beams of span less than 10 m, to satisfy the vertical deflection criteria,

$$\frac{Effective\ span}{Effective\ depth} \leq 10\ \times Modification\ factor(MF)$$

*Assuming

(MF = 1, and

Effective span = c/c distance between supports)

$$\therefore \frac{6000}{d} \leq 20$$

$$d \geq \frac{6000}{20}$$

$$d \geq 300\ mm$$

Taking $d{\sim}350\ mm \therefore D = 350 + 50 = 400\ mm$

As per [22.2(a), IS 456:2000] for simply supported beams,

Effective span (l) = min(c/c distance between supports, clear span+$d/2$)

$$\therefore l = \min\,(6, 6 - 0.25 + 0.35) = \min\,(6, 6.1) = 6\ m$$

Hence,

$$d = 350\ mm, D = 400\ mm, l = 6\ m$$

Step#2 Calculation of the factored maximum bending moment acting on the beam i.e., the design bending moment.

Calculation of loads: **[2]**

LL = 10 kN/m

DL = 5 kN/m

Self-Weight = $25\ \times b \times D = 25 \times 0.25 \times 0.4 = 2.5$ kN/m

Total factored load intensity acting on the beam(w) = $1.5 \times (10 + 5 + 2.5) = 26.25$kN/m

Calculation of maximum/design bending moment:

For a simply-supported beam maximum bending moment acts at the mid-span section,

$$\therefore\ M_u = \frac{wl^2}{8} = \frac{26.25 \times 6^2}{8} = 118.125 \text{ kN-m}$$

Step#3 Calculation of the required effective depth of the section. **[5]**

The beam is generally designed as an under-reinforced beam

$$\therefore M_u < M_{u,limt}$$

$$\therefore 118.125 \times 10^6 < 0.36 f_{ck} b x_{u,limit} \times \left(d - 0.42 x_{u,limit}\right)$$

For Fe 415 $x_{u,limit} = 0.48\ d$

$\therefore 118.125 \times 10^6 < 0.36 \times 25 \times 250 \times 0.48d \times (d - (0.42 \times 0.48d))$

$$\therefore d > \sqrt{\frac{118.125 \times 10^6}{0.36 \times 25 \times 250 \times 0.48 \times (1 - (0.42 \times 0.48))}}$$

$\therefore d > 370.125\ mm$

$\because d_{provided} = 350\ mm < d_{required} = 370.125\ mm$

Taking $d \sim 400\ mm \therefore D = 400 + 50 = 450\ mm$

As per [22.2(a), IS 456:2000] for simply supported beams,

Effective span (l) = min(c/c distance between supports, clear span$+d/2$)

$\therefore l = \min(6, 6 - 0.25 + 0.4) = \min(6, 6.15) = 6\ m$

Hence,

$d = 400\ mm, D = 450\ mm, l = 6\ m$

Recalculating for revised d and D

Self-Weight = $25 \times b \times D = 25 \times 0.25 \times 0.45 = 2.8125$ kN/m

Total factored load intensity acting on the beam(w) = $1.5 \times (10 + 5 + 2.8125) = 26.72$kN/m

Calculation of maximum/design bending moment:

For a simply-supported beam maximum bending moment acts at the mid-span section,

$\therefore M_u = \frac{wl^2}{8} = \frac{26.72 \times 6^2}{8} = 120.24$ kN-m

The beam is generally designed as an under-reinforced beam

$\therefore M_u < M_{u,limt}$

$\therefore 120.24 \times 10^6 < 0.36 f_{ck} b x_{u,limit} \times (d - 0.42 x_{u,limit})$

For Fe 415 $x_{u,limit} = 0.48\ d$

$\therefore 120.24 \times 10^6 < 0.36 \times 25 \times 250 \times 0.48d \times (d - (0.42 \times 0.48d))$

$$\therefore d > \sqrt{\frac{120.24 \times 10^6}{0.36 \times 25 \times 250 \times 0.48 \times (1 - (0.42 \times 0.48))}}$$

$\therefore d > 373.424\ mm$

$\because d_{provided} = 400\ mm > d_{required} = 373.424\ mm \therefore$ OK

<u>**Step#4** Calculation of the area of tension steel required.</u> **[5]**

For an under-reinforced section,

For equilibrium

$M_u = C \times Z = T \times Z$

$\therefore 120.24 \times 10^6 = 0.87 f_y A_{st} \times (d - 0.42 x_u)$

(Where,

From equilibrium $C = T$

$\therefore 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$\therefore x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$)

$$\therefore 120.24 \times 10^6 = 0.87 f_y A_{st} \times (d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b})$$

$$\therefore 120.24 \times 10^6 = 0.87 \times 415 \times A_{st} \times (400 - \frac{0.42 \times 0.87 \times 415 \times A_{st}}{0.36 \times 25 \times 250})$$

$$\therefore 333028.6664 = 400 A_{st} - 0.067396 {A_{st}}^2$$

On solving $A_{st,required} = 1001.602\ mm^2$

Let providing 2-20 mm dia. bars and 1-25 mm dia. bar as the main tension reinforcement,

$$\therefore A_{st,provided} = \left(2 \times \frac{\pi}{4} \times 20^2\right) + \left(\frac{\pi}{4} \times 25^2\right) = 1119.193\ mm^2$$

$$\therefore p_{t,provided} = \frac{A_{st,provided}}{bd} \times 100\ \% = \frac{1119.193}{250 \times 400} \times 100\ \% = 1.12\ \%$$

Checks:

#1 Over-reinforced section,

$$x_{u,limit} = 0.48d = 0.48 \times 400 = 192\ mm$$

$$x_u = \frac{0.87 \times 415 \times 1119.193}{0.36 \times 25 \times 250} = 179.95\ mm$$

$\because x_u < x_{u,limit}$ the designed section is an under-reinforced section

#2 $A_{st,min}$

$$A_{st,min} = \frac{0.85 \times bd}{f_y} = \frac{0.85 \times 250 \times 400}{415} = 204.82\ mm^2$$

$\because A_{st,provided} > A_{st,min}$ ∴Ok

#3 $A_{st,max}$

$$A_{st,max} = 0.04bD = 0.04 \times 250 \times 450 = 4500\ mm^2$$

$\because A_{st,provided} < A_{st,max}$ ∴Ok

#4 l/d ratio

$$f_s = 0.58 f_y \frac{A_{st,required}}{A_{st,provided}} = \frac{0.58 \times 415 \times 1001.602}{1119.193} = 215.41\ N/mm^2$$

$$p_{t,provided} = 1.12\ \%$$

From [Fig. 4 IS 456:2000]

Modification factor (MF) ~1.05

$$\frac{l}{d} < 20 \times 1.05$$

$$\therefore d > \frac{l}{20 \times 1.05}$$

$$\therefore d > \frac{6 \times 1000}{20 \times 1.05}$$

$\therefore d > 285.714\ mm$

$\because d_{provided} > d_{required}$ $\therefore$ OK

Step#5 Reinforcement details at the critical section. **[5]**

*Assuming dia. of stirrups as 8 mm. and maximum size of coarse aggregate as 20 mm.

Minimum clear cover for moderate exposure = 30 mm [Table 16 IS 456:2000]

Calculation of horizontal spacing:

$$S = \frac{250 - 60 - 16 - 40 - 25}{2} = 54.5\ mm$$

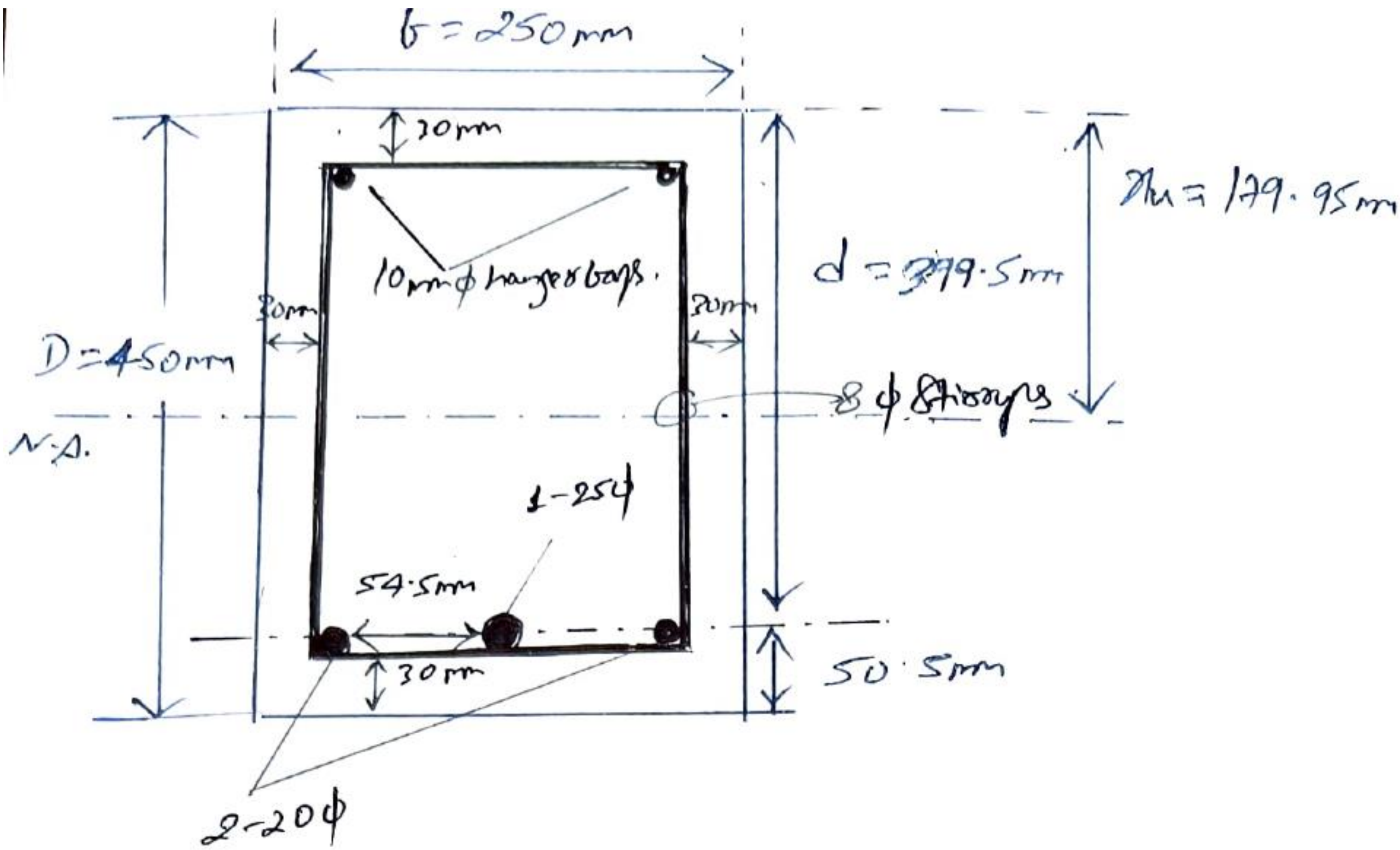
Checks:

$S_{\min=}$ max $\left(25, (5 + 20)\right) = 25\ mm$ [26.3.2(a) IS 456:2000]

$S_{\max=}$ 180 mm [Table 15 IS 456:2000] *Assuming no redistribution

$\because S$ is within the permissible range $\therefore$ OK

Reinforcement details,



Assuming that the c.g. of reinforcement group passes through the center of 25 mm bar,

$\therefore$ Actual effective cover $= 30 + 8 + \frac{25}{2} = 50.5$ mm

$\therefore$ Actual effective depth, $d = 450 - 50.5 = 399.5\ mm \sim 400\ mm$ $\therefore$ OK

A.3: Sol:

$f_{ck} = 25 N/mm^2$; $f_y = 415\ N/mm^2$; $b = 250\ mm$; $D = 400\ mm$; c/c distance between supports = 6 m

Step#1 Calculation of the trial effective depth, and trial effective span of the beam. **[1.25]**

Taking effective cover = 50 mm

$\therefore d = 400 - 50 = 350\ mm$

As per [22.2(a), IS 456:2000] for simply supported beams,

Effective span (l) = min(c/c distance between supports, clear span+$d/2$)

$\therefore l = \min(6, 6 - 0.25 + 0.35) = \min(6, 6.1) = 6\ m$

Hence,

$d = 350\ mm, D = 400\ mm, l = 6\ m$

Step#2 Calculation of the factored maximum bending moment acting on the beam i.e., the design bending moment.

Calculation of loads: **[1.25]**

LL = 10 kN/m

DL = 5 kN/m

Concentrated DL = 30 kN

Self-Weight = $25 \times b \times D = 25 \times 0.25 \times 0.4 = 2.5$ kN/m

Total factored load intensity acting on the beam(w) = $1.5 \times (10 + 5 + 2.5) = 26.25$ kN/m

Total factored concentrated load acting at the mid-span (p) = $1.5 \times 30 = 45$ kN/m

Calculation of maximum/design bending moment:

For a simply-supported beam maximum bending moment acts at the mid-span section,

$\therefore M_u = \frac{wl^2}{8} + \frac{pl}{4} = \frac{26.25\times6^2}{8} + \frac{45\times6}{4} = 185.63$ kN-m

Step#3 Check whether a doubly reinforced section is required or not. **[2.5]**

$M_{u,limit} = C_{limit} \times Z_{limit}$

$C_{limit} = 0.36 f_{ck} b x_{u,max}$

$Z_{limit} = d - 0.42 x_{u,max}$

For f_y= 415 N/mm^2 , $x_{u,max} = 0.48d$

$\therefore x_{u,max} = 0.48 \times 350 = 168$ mm

$$M_{u,limit} = \left(0.36 f_{ck} b x_{u,max}\right) \times \left(d - 0.42 x_{u,max}\right)$$
$$= \left[(0.36 \times 25 \times 250 \times 168) \times \left(350 - (0.42 \times 168)\right)\right] \times 10^{-6}\ kN - m$$
$$= 105.63\ kN - m$$

$\because M_u = 185.63\ kN - m > M_{u,limit} = 105.63\ kN - m$

Hence a doubly reinforced section is required

Step#4 Calculation of the area of tension steel required corresponding to $M_{u,limit}$. **[2.5]**

$M_{u,limit} = T_{limit} \times Z_{limit}$

$T_{limit} = 0.87 f_y A_{st,limit} = 0.87 \times 415 \times A_{st,limit}$

$Z_{limit} = d - 0.42 x_{u,limit} = 350 - 0.42 \times 168$

$\therefore M_{u,limit} = 0.87 \times 415 \times A_{st,limit} \times (350 - 0.42 \times 168)$

$105.63 \times 10^6 = 0.87 \times 415 \times A_{st,limit} \times (350 - 0.42 \times 168)$

$A_{st,limit} = \frac{105.63\times10^6}{0.87\times415\times(350-0.42\times168)} = 1047$ mm²

Step#4 Calculation of the area of tension steel required corresponding to $M_u - M_{u,limit}$: **[2.5]**

$M_u - M_{u,limit} = T_2 \times Z_2$

$T_2 = 0.87 \times f_y \times A_{st2}$

$Z_2 = d - d'$

Assuming $d' = 50$ mm

$\therefore (185.63 - 105.63) \times 10^6 = 0.87 \times 415 \times A_{st2} \times (350 - 50)$

$\therefore A_{st2} = \frac{(185.63-105.63)\times10^6}{0.87\times415\times(350-50)} = 738.59$ mm²

Step#5 Calculation of the total area of tension steel required **[5]**

$A_{st} = A_{st,limit} + A_{st2}$

$\therefore A_{st} = 1047 + 738.59 = 1785.59$ mm²

Let providing 28 mm dia. bars

$\therefore no.of\,bars = \frac{1785.59}{\frac{\pi}{4}\times28^2} = 2.89 \sim 3$ bars

$A_{st,provided} = 3 \times \frac{\pi}{4} \times 28^2 = 1847.26$ mm²

*Assuming dia. of stirrups as 8 mm. Minimum clear cover for moderate exposure = 30 mm [Table 16 IS 456:2000]

$\therefore$ actual effective cover $= 30 + 8 + \frac{28}{2} = 52$ mm

$\therefore$ actual effective depth, $d = 400 - 52 = 348$ mm

Recalculating for revised d

Effective span (l) = min(c/c distance between supports, clear span+$d/2$)

$\therefore l = \min(6, 6 - 0.25 + 0.348) = \min(6, 6.098) = 6\,m$

$x_{u,max} = 0.48 \times 348 = 167.04$ mm

$$M_{u,limit} = \left(0.36 f_{ck} b x_{u,max}\right) \times \left(d - 0.42 x_{u,max}\right)$$
$$= \left[(0.36 \times 25 \times 250 \times 167.04) \times \left(348 - (0.42 \times 167.04)\right)\right] \times 10^{-6}\,kN - m$$
$$= 104.43\,kN - m$$

$M_u = 185.63$ kN-m (as no change in effective length and loads).

$\therefore A_{st,limit} = \frac{M_{u,limit}}{0.87\times415\times(d-0.42x_{u,max})} = \frac{104.43\times10^6}{0.87\times415\times(348-0.42\times167.04)} = 1041.02$ mm²

$M_u - M_{u,limit} = 185.63 - 104.43 = 81.2\,kN - m$

$\therefore A_{st2} = \frac{M_u - M_{u,limit}}{0.87\times f_y\times(d-d')} = \frac{81.2\times10^6}{0.87\times415\times(348-50)} = 754.697$ mm²

$A_{st} = A_{st,limit} + A_{st2} = 1041.02 + 754.697 = 1795.717$ mm²

$\because A_{st,provided} = 1847.26\,mm^2 > A_{st,required} = 1795.717\,mm^2$ ∴OK

Hence, Providing

$A_{st,limit} = A_{st,limit,required} = 1041.02$ mm²

$\therefore A_{st2,provided} = A_{st,provided} - A_{st,limit\ provided} = 1847.26 - 1041.02 = 806.24$ mm²

$A_{st,provided} = 1847.26$ mm²

Checks:

#1 $A_{st,min}$

$$A_{st,min} = \frac{0.85 \times bd}{f_y} = \frac{0.85 \times 250 \times 348}{415} = 178.2\ mm^2$$

$\because A_{st,provided} > A_{st,min}$ ∴OK

#2 $A_{st,max}$

$$A_{st,max} = 0.04bD = 0.04 \times 250 \times 400 = 4000\ mm^2$$

$\because A_{st,provided} < A_{st,max}$ ∴OK

Step#6 Calculation of the total area of compression steel required **[5]**

For equilibrium

$C_2 = T_2$

$C_2 = (f_{sc} - f_{cc}) \times A_{sc}$

For Fe 415, $\frac{d'}{d} = \frac{50}{348} = 0.144$

From interpolation, $f_{sc} = \left(\frac{342.4-351.9}{0.15-0.10} \times (0.144 - 0.1)\right) + 351.9 = 343.5$ MPa

$T_2 = 0.87 \times f_y \times A_{st2}$

$\therefore (f_{sc} - f_{cc}) \times A_{sc} = 0.87 \times f_y \times A_{st2}$

$A_{sc} = \frac{0.87 \times f_y \times A_{st2,provided}}{(f_{sc} - f_{cc})} = \frac{0.87 \times 415 \times 806.24}{343.5-(0.45 \times 25)} = 876.126$ mm^2

Let providing 20 mm dia. bars

$\therefore no.of\,bars = \frac{876.126}{\frac{\pi}{4} \times 20^2} = 2.79 \sim 3$ bars

$A_{sc,provided} = 3 \times \frac{\pi}{4} \times 20^2 = 942.48$ mm^2

*Assuming dia. of stirrups as 8 mm. Minimum clear cover for moderate exposure = 30 mm [Table 16 IS 456:2000]

∴ actual effective cover to compression reinforcement, $d' = 30 + 8 + \frac{20}{2} = 48$ mm

Recalculating for revised d'

$A_{st2,required} = \frac{M_u - M_{u,limit}}{0.87 \times f_y \times (d-d')} = \frac{81.2 \times 10^6}{0.87 \times 415 \times (348-48)} = 749.67$ mm^2

$\because A_{st2,provided} = 806.24\ mm^2 > A_{st2,required} = 749.67\ mm^2$ ∴OK

$A_{st,required} = 749.67 + 1041.02 = 1790.68$ mm^2

$\because A_{st,provided} = 1847.26\ mm^2 > A_{st,required} = 1790.68\ mm^2$ ∴OK

$$\frac{d'}{d} = \frac{48}{348} = 0.138$$

From interpolation, $f_{sc} = \left(\frac{342.4-351.9}{0.15-0.10} \times (0.138 - 0.1)\right) + 351.9 = 344.68$ MPa

$A_{sc,required} = \frac{0.87 \times f_y \times A_{st2,provided}}{(f_{sc} - f_{cc})} = \frac{0.87 \times 415 \times 806.24}{344.68-(0.45 \times 25)} = 873.026$ mm^2

$\because A_{sc,provided} = 942.48\ mm^2 > A_{sc,required} = 873.026 mm^2$ ∴OK

Checks:

#1 $A_{sc,max}$

$A_{st,max} = 0.04bD = 0.04 \times 250 \times 400 = 4000\ mm^2$

$\because A_{sc,provided} < A_{st,max}$ ∴OK

Step#7 Reinforcement details at the critical section. **[5]**

*Assuming dia. of stirrups as 8 mm., dia. of hanger bars as 10 mm., and maximum size of coarse aggregate as 20 mm.

Minimum clear cover for moderate exposure = 30 mm [Table 16 IS 456:2000]

Calculation of horizontal spacing for tension steel:

$$S_t = \frac{250 - 60 - 16 - (3 \times 28)}{2} = 45\ mm$$

Calculation of horizontal spacing for compression steel:

$$S_c = \frac{250 - 60 - 16 - (3 \times 20)}{2} = 57\ mm$$

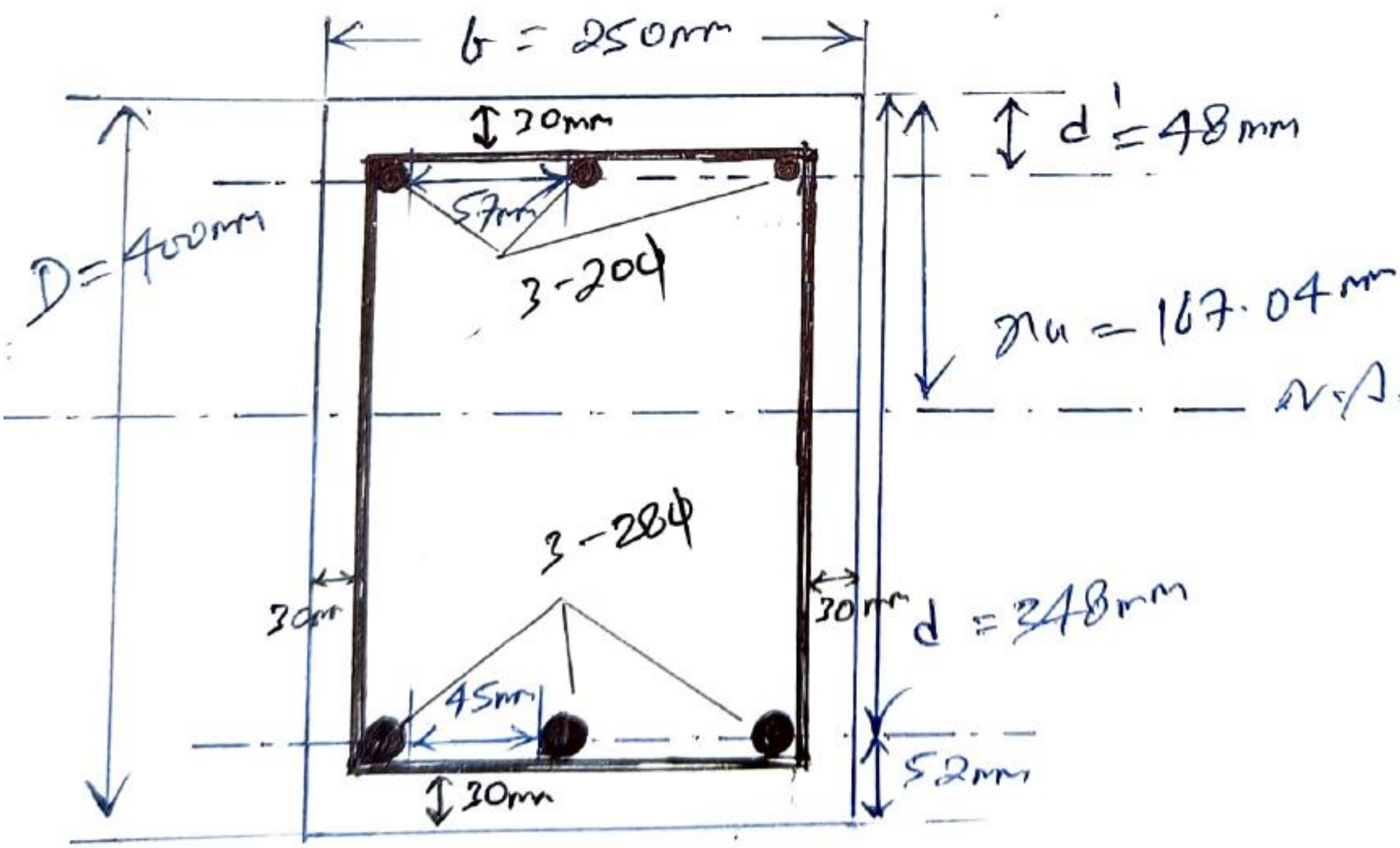
Checks:

$S_{\text{min}=}$ max $\left(28, (5 + 20)\right) = 28\ mm$ [26.3.2(a) IS 456:2000]

$S_{\text{max}=}$ 180 mm [Table 15 IS 456:2000] *Assuming no redistribution

$\because$ both S_t and S_c are within the permissible range ∴ OK

Reinforcement details,



Step#8 l/d ratio. **[5]**

Modification factor for tension steel,

$$f_s = 0.58 f_y \frac{A_{st,required}}{A_{st,provided}} = \frac{0.58 \times 415 \times 1790.68}{1847.26} = 233.33\ N/mm^2$$

$$p_{t,provided} = \frac{A_{st,provided}}{bd} \times 100\ \% = \frac{1847.26}{250 \times 348} \times 100\ \% = 2.12\ \%$$

∴ From [Fig. 4 IS 456:2000], Modification factor (MF_t) ~ 0.85

Modification factor for compression steel,

$$p_{c,provided} = \frac{A_{sc,provided}}{bd} \times 100\ \% = \frac{942.48}{250 \times 348} \times 100\ \% = 1.083\ \%$$

∴ From [Fig. 5 IS 456:2000], Modification factor (MF_c) ~ 1.25

Hence, as per [23.2.1, IS 456:2000] for simply supported beams of span less than 10 m, to satisfy the vertical deflection criteria

$$\frac{l}{d} < 20 \times 0.85 \times 1.25$$

$$\therefore d > \frac{l}{20 \times 0.85 \times 1.25}$$

$$\therefore d > \frac{6 \times 1000}{20 \times 0.85 \times 1.25}$$

$$\therefore d_{required} > 282.6\ mm$$

$$\because d_{provided} > d_{required}\ \therefore \text{OK}$$

A.4: Sol:

Step#1 Calculation of the effective width of the flange. **[3]**

For an isolated T beam,

Effective width, $b_f = min\left(\frac{l_o}{l_o/b+4} + b_w\ ,\ b\right) = min\left(\frac{6000}{6000/1000+4} + 230\ ,\ 1000\right) = \min(830,1000) = 830 \text{ mm}$

Step#2 Assuming that the N.A. lies in the flange, i.e., $x_u < D_f$ **[6]**

$\because$ the N.A. lies in the flange, i.e., $x_u < D_f$

$\therefore$ considering the beam as a rectangular beam of width b_f

For equilibrium at the limit state of collapse,

$C = T$

$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

Min. grade of concrete for mild exposure: M 20 (Table 5 IS 456:2000)

$$\therefore x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{\left(0.87 \times 415 \times 6 \times \frac{\pi}{4} \times 20^2\right)}{0.36 \times 20 \times 830} = 113.883\ mm$$

$\because\ x_u (= 113.883\ mm) > D_f (= 110\ mm)$, *our assumption was incorrect*, N.A. lies in the web.

Step#3 Assuming that the N.A. lies in the web, i.e., $x_u > D_f$ **[10]**

Trial#1

$$\frac{D_f}{d} = \frac{110}{520} = 0.212 > 0.2$$

*Assuming

a) the section is an under-reinforced section, i.e., $x_u < x_{u,max}$, so that $T = 0.87 f_y A_{st}$

b) $\frac{D_f}{x_u} > 0.43$

$\therefore$ as per [G-2.3 IS 456:2000]

For equilibrium at the limit state of collapse,

$C = T$

$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$

(Where,

$y_f = 0.15 x_u + 0.65 D_f = 0.15 x_u + (0.65 \times 110) = 0.15 x_u + 71.5)$

$\therefore 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w)(0.15 x_u + 71.5) = 0.87 f_y A_{st}$

$$(0.36 \times 20 \times 230 \times x_u) + \left(0.45 \times 20 \times (830 - 230) \times (0.15 x_u + 71.5)\right) = 0.87 \times 415 \times 6 \times \frac{\pi}{4} \times 20^2$$

$1656 x_u + 810 x_u + 386100 = 680563.2165$

$\therefore x_u = \frac{680563.2165 - 386100}{1656 + 810} = 119.41 \text{ mm}$

Checks:

(i) $x_u = 119.41 \text{ mm} > D_f = 110\ mm$, *our assumption was correct*, N.A. lies in the web.

(ii) $x_{u,max} = 0.48d$

$d = 600 - 80 = 520 \text{ mm}$

$\therefore\ x_{u,max} = 0.48d = 0.48 \times 520 = 249.6 \text{ mm},$

$\because\ x_u < x_{u,max}$, so $T = 0.87 f_y A_{st}$ t

(iii) $\frac{D_f}{x_u} = \frac{110}{119.41} = 0.96 > 0.43$

$\because x_{u,max} > x_u > D_f$

$\therefore$ *Our assumptions were correct* we will work on y_f instead of D_f

Step#4 Calculation of M_{u-} **[6]**

$$y_f = 0.15x_u + 71.5 = (0.15 \times 119.41) + 71.5 = 89.4115 \text{ mm}$$

$$M_u = [(0.36 f_{ck} b_w x_u) \times (d - 0.42x_u)] + \left[0.45 f_{ck}(b_f - b_w) y_f \times \left(d - \frac{y_f}{2}\right)\right]$$

$$= \left[0.36\frac{x_u}{d}\left(1 - 0.42\frac{x_u}{d}\right) f_{ck} b_w d^2\right] + \left[0.45 f_{ck}(b_f - b_w) y_f \times \left(d - \frac{y_f}{2}\right)\right]$$

$$= \left\{\left[0.36 \times \frac{119.41}{520}\left(1 - \frac{0.42 \times 119.41}{520}\right) \times 20 \times 230 \times 520^2\right] + \left[0.45 \times 20 \times (830 - 230) \times 89.4115 \times \left(520 - \frac{89.4115}{2}\right)\right]\right\} \times 10^{-6} \text{ kN-m}$$

= 322. 39 kN-m