BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE
MID-SEMESTER EXAMINATION (1st SEMESTER) 2023-2024
NUMERICAL ANALYSIS - CE F324 (OPEN BOOK)

Dated: 10.10.2023

Max. Marks: 50 **Max. Duration: 90 minutes**

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1. The groundwater flow through a 2D porous system is described by the given PDE for pressure head (h) distribution in a steady state. Consider a porous domain size of 10 m x 10 m, Dirichlet boundary condition with the head of 1m, 2m, 2m, and 1m at left, right, top, and bottom boundaries respectively.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

a) What is the type of PDE? Write the Finite difference (FD) equation for the above equation

b) Express the above FD equation so that it can be used as the Liebmann/iterative method and determine the head distribution by this method by employing over-relaxation of 1.5 in the solver. What will be the effect of over-relaxation on the solver? **(10)**

2. Consider the following PDE which represents chemical concentration distribution in a reactor or contaminant transport through a river in a non-conservative system.

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - U\frac{\partial C}{\partial x} - kC$$

a) What is the type of PDE and the physical significance of each term?

b) Formulate the Finite Difference equation by explicit and implicit methods.

c) Which of the above formulae would create a stability problem and find the stability condition, if any? **(10)**

3. Develop the Crank-Nicholson Finite Difference formulation for the 1D unsteady heat diffusion equation given below. Mention the type of PDE.

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial x^2}$$

Perform the stability check by perturbation/Von Neumann Stability theory (consider sinusoidal perturbation) and find the stability criteria for the above FD formulation. **(10)**

4. Consider 1D unsteady Advection-diffusion equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - U\frac{\partial C}{\partial x}$$

Discuss and derive the condition of numerical diffusion. Define the Peclet number condition in the above equation. Also, Find the FD formulation under which the Courant number condition will arise. **(10)**

6. Glucose infusion in the human body through the bloodstream is a significant and very common process. If we consider the amount of glucose in the bloodstream of a patient at any time instant t is G(t) and glucose is infused at a constant rate of k millimoles/litre in a minute, such that simultaneously the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present, then the function G(t) satisfies the following mathematical model

$$\frac{\partial G}{\partial t} = 0.5 - 0.11G$$

The initial value of glucose present in the bloodstream is 3.5 millimoles/litre. Use the best possible order of numerical method for ODE to find the distribution of glucose with time. Take a time step of 1 min. **(10)**