

Q.1. Government operated flood relief camp operates with three check-out counters to distribute food. The manager is currently using the following schedule to determine the number of counters in operation, depending on the number of people entering the camp:

Number of people in camp	Number of counters in operation
1 to 3	1
4 to 6	2
More than 6	3

Analyze two independent cases (A & B) given below:

A. Suppose that the interarrival time at the check-out area is exponential with mean 5 minutes and that the checkout time per person is also exponential with mean 10 minutes. Suppose that the camp adds a fourth counter and that counters will open based on increments of two persons. Determine the following: **[3 M]**

- The steady-state probabilities, p_n for all n .
- The probability that a fourth counter will be needed
- The average number of idle counters.

B. Now, suppose that all three counters are always open, and that the operation is set up such that the person will go to the first empty counter. Determine the following: **[3 M]**

- The probability that all three counters will be in use
- The probability that an arriving person will not wait.

Q.2. Car service center employs four specialist mechanics in the engine services area. Not all the cars arriving for the service require services in the engine and only those requiring some tuning or de-carbonizing are sent to the engine repair department. The mechanics can service one car engine at an average time of two hours. The cars are registered at a single point and then directed to any of the four mechanics, in case there is an engine-related problem. The cars requiring engine repair services arrive at an average rate of 1.5 cars per hour. Determine the following: **[6 M]**

- The utilization factor
- Probability that the system shall be idle
- Probability that there will be four cars requiring engine services
- Probability that there will be eight cars requiring engine services
- Expected number of cars waiting in the queue for engine repairs
- Expected number of cars in the queue and receiving service in the engine department
- Average waiting time in the queue in the engine repair area
- Average time being spent by the car in waiting for engine service and in getting engine serviced

Q.3. The advertising alternatives for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in Table below.

Attribute	Television	News paper	Radio
Cost per advertisement (\$)	2000	600	300
Audience per advertisement	100,000	40,000	18,000

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is \$18,200. How many advertisements should be run in each of the three types of media to maximize the total audience? Solve the problem using Simplex method. **[6 M]**

Q4. Three crops are to be grown in an available land area of 250 hectare (ha) in a particular season. Gross benefits from each of the crops per hectare are Rs. 2000, Rs 2500, Rs 1000 respectively. Total available water from surface and ground water sources is 4 ha-m. Crops water requirements for each crop per hectare are 3 cm, 2 cm and 1 cm respectively. Water charges per hectare are Rs. 200, Rs. 120 and Rs. 140 respectively. Cost of seeds and fertilizer per hectare are Rs. 200, Rs. 120 and Rs. 140. Land preparation and other charges per hectare are Rs 100, Rs 150 and Rs 125. Based on demography considerations, lower and upper limits for each of the crops are fixed as 40 and 90 ha; 45 and 100 ha; 45 and 60 ha respectively. Formulate the problem in linear programming framework for maximization of net benefits. **[6 M]**

Details of given data (Rs/ha) for each crop

Crop no.	Gross benefits	Cost of seeds and fertilizers	Water charges	Land preparation and other charges	Net benefits
1	2000	400	200	100	1300
2	2500	500	120	150	1730
3	1000	200	140	125	535

Q5. Consider the following LPP,

[6 M]

$$\text{Maximize } z = 3x_1 + 2x_2 + 3x_3$$

Subjected to

$$2x_1 + x_2 + x_3 \leq 2$$

$$3x_1 + 4x_2 + 2x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0$$

- Use two phase method and obtain the optimal simplex table at the end of Phase I
- Will be necessary to carry out phase II, if yes, please do the same. If it is not necessary, please state the reason.
- Obtain the optimal value of z and corresponding values of all decision variables.