

1. (a) What is the difference between natural frequency and fundamental frequency? [10]
 (b) Write the difference between an explicit integration scheme and an implicit integration scheme.
 (c) Prove that mode shapes are orthogonal with respect to stiffness and mass matrices.
 (d) What is the difference between time-domain analysis and frequency-domain analysis?
 (e) What is Rayleigh damping? Show the relation between the modal damping ratio and natural frequency via diagram when damping is proportional to both mass and stiffness.
2. An SDOF system with $k/m = 100$, $m = 2$ kg, and $\zeta = 5\%$ is subjected to a periodic loading as shown in Fig. 1. Find the response of the system by taking the only first term of the Fourier series of the loading. Consider $T_0 = 0.8$. [7]
3. Determine the response ($x(t)$) of an undamped SDOF system with mass m and natural period T_n subjected to a rectangular pulse shown in Fig. 2. [5]
4. A massless frame having two members is shown in Fig. 3. The material properties, geometric dimensions and lumped masses are assigned to the members. Formulate the free vibration equation for the frame in terms of translational DOFs u_1 and u_2 . Axial and shear deformations are neglected. [Hint: Use flexibility influence coefficients approach] [5]
5. The two-story shear frame along with the stiffness of columns and lumped masses ($k/m=100$; $m=2$ kg) is shown in Fig. 4. Answer the following questions. (a) Find the frequencies and corresponding mode shapes of the frame. Also, draw the mode shapes. (b) Construct the Rayleigh damping matrix by assuming the modal damping ratio $\zeta = 5\%$. (c) Find the values of displacement, velocity, and acceleration of the frame at $t = 2$ sec using Newmark's method based on average acceleration. The time step (Δt) is taken as 0.02 sec. The displacement, velocity, and acceleration of the frame at $t = 2$ sec are $u^T = [0.5 \ 0.15]$; $\dot{u}^T = [-0.12 \ 0.24]$; $\ddot{u}^T = [1.4 \ 2.6]$; respectively and loading at $t = 2.02$ sec is $P^T(2.02s) = [3 \ -1]$. [4+3+6]

