BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2022– 2023
Advanced Structural Mechanics and Stability (CE G552) – Comprehensive Examination

Duration: 3 HRS. (Closed book) **Max. Marks: 40** **Date: 21-12-2022**

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1. (i) What do you mean by the failure of the structures? **[10]**
(ii) What is warping and why warping is zero for the circular section when it is subjected to torsion?
(iii) Describe the plane of material symmetry. Orthotropic material exhibits how many planes of material symmetry?
(iv) What is stability and explain the stability of structures via column example?
(v) What is buckling and explain the difference between the bifurcation point and limit point?

2. The stress components at a material point in a body for the case of a deviatoric state of stress are given as, $\sigma_{xx} = \sigma_{zz} = c$ MPa and $\tau_{xy} = \tau_{yz} = \tau_{zx} = 2c$ MPa. Determine the value of the constant c based on (i) Maximum shear stress theory, and (ii) Maximum distortion energy theory. Use a factor of safety equal to 2 and the body is made of ductile steel whose elastic (yield) limit is 210 MPa. The modulus of elasticity (E) and rigidity (G) is 200 GPa and 80 GPa respectively. **[6]**

3. The warping function is given as, $\psi = -y$ for a bar subjected to torque T. The given warping function satisfies both the governing equation and boundary condition. Prove that the given warping function represents the shape of the bar as a circular profile. If the radius of the circle is considered to be unity, then answer the following questions, **(a)** Find the polar moment of inertia (J). **(b)** Find the angle of twist (θ) of a circular bar in terms of modulus of rigidity (G) and torque (T). **(c)** Find the maximum shear stress $(\tau_{\max})$ in terms of torque (T) . **[6]**

4. A thin-walled box section has two cells is subjected to torque ***T*,** as shown in **Fig.1**. The left closed cell is made of half-circular and half-rectangular sections. The right closed cell is an open square cell. Both the cells have constant wall thickness ***t***. Find the shear stress in each cell and the angle of twist per unit length of the box section. The modulus of rigidity is G **[6]**

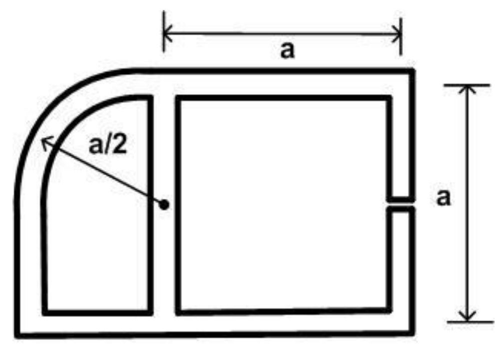

Fig. 1

5. The elastic stiffness matrix ($\mathbf{C_{ijkl}}$) of tetragonal material having six independent constants is given below. Derive the elastic stiffness matrix ($\mathbf{C_{ijkl}}$) of isotropic materials in terms of two independent constants from six independent constants. Finally, write the relation between elastic constants. **[6]**

$$\mathbf{C_{ijkl}} = \begin{bmatrix} \mathbf{C_{1111}} & \mathbf{C_{1122}} & \mathbf{C_{1122}} & 0 & 0 & 0 \\ & \mathbf{C_{2222}} & \mathbf{C_{2233}} & 0 & 0 & 0 \\ & & \mathbf{C_{2222}} & 0 & 0 & 0 \\ & & & \mathbf{C_{2323}} & 0 & 0 \\ \text{Sym} & & & & \mathbf{C_{1212}} & 0 \\ & & & & & \mathbf{C_{1212}} \end{bmatrix}$$

6. Find the matrices A, B, and D of the laminate. The laminate is formed by three laminas (see **Fig. 2**). Thickness of each lamina is 2 mm. The fiber directions in all the laminas coincide with the *x*-axis (i.e., $\mathbf{0^0}$). The *z*-axis is along the thickness direction. The material properties of the lamina along the principal material direction are as follows, $E_{11} = 200$ GPa; $E_{22} = E_{11}/25$; $G_{12} = 0.5E_{22}$; $\nu_{12} = 0.3$ **[6]**

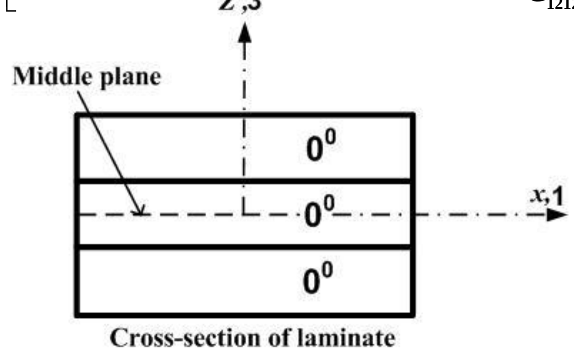

Fig. 2