

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
 First Semester (2017-2018), Comprehensive Examination
 Course: Finite Element Analysis (CE G619)

Date: 11th Dec. 2017 (Room:2201)

Max. Marks: 70

Duration: 2:00PM-5:00PM

Q.1. Find the approximate solution of the partial differential equation given below by (i) Collocation method(Collocation point (0.5, 0.5)) (ii) Galerkin method (iii) Least Square method (iv) Ritz method (weak form). [15]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 80, \quad 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \text{ with conditions, } u(0,0) = 0 \text{ and } u(1,1) = 80$$

Find the values of u at $x = 0.5$ and $y = 0.5$ in all methods. Assume the solution to be $u = C_1 \phi_1 + \phi_0$, take $\phi_0(x, y) = 80xy$ and $\phi_1(x, y) = x(x - 1) + y(y - 1)$.

Q.2. In the quadrilateral domain shown in **Fig.1**, the following differential equation with a single field variable(u) is valid,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} - 5 = 0, \quad u(0,0) = 0 \text{ and } u(3.0,4.0) = 0,$$

Find the weak form of the equation. Taking the whole domain as one element find the value of the field/primary variable(u) at (0, 2.15), (2.5,0.15) and (0.5,0.5). Map the element in natural co-ordinate(ξ - η) system and use isoparametric formulation. [20]

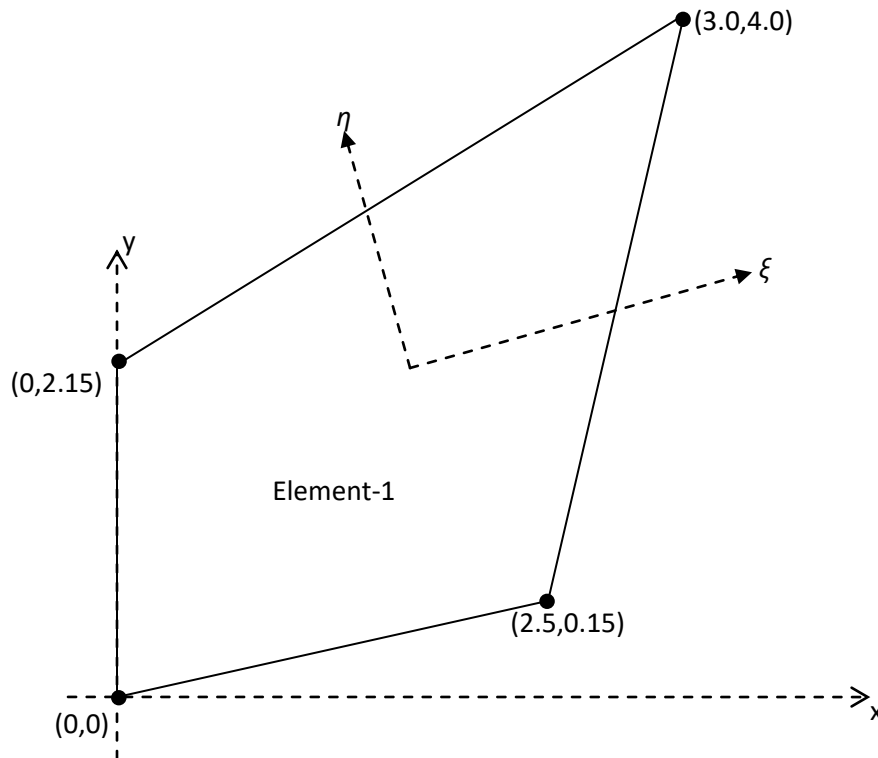


Fig.1 Quadrilateral domain

Q.3. What is the difference between Euler-Bernoulli beam theory and Timoshenko beam theory. Derive the governing differential equations for beam bending. Find the weak form of the equations. Taking an element(**Fig.2**), find the elemental equilibrium equation(finite element model) using both theories. What is shear locking. [20]

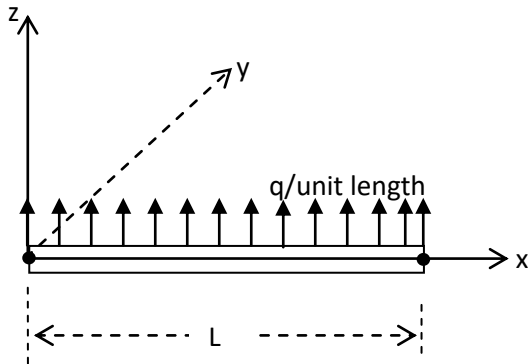


Fig.2 Beam element

Q.4. Find the displacement at D and reactions at A, B and C of the 2-D truss given in **Fig.3**. The lengths and cross-sectional areas of all members are 1.5m and 4000mm² respectively. Take E=201 GPa. [10]

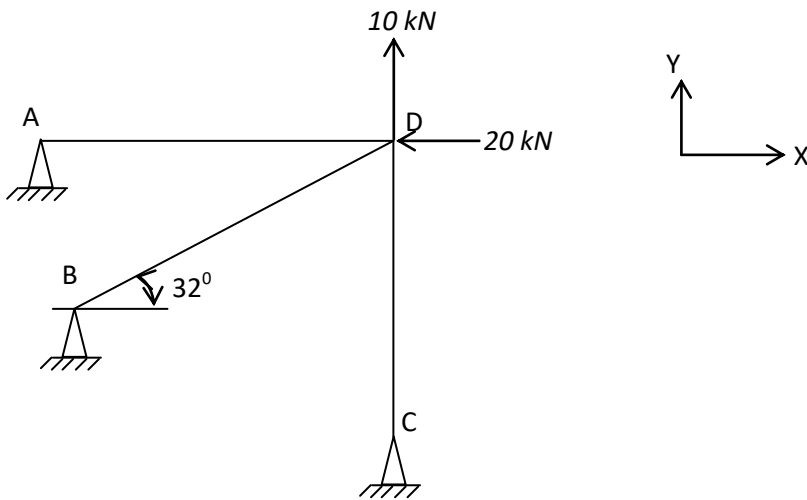


Fig.3 2-D Truss

Q.5. Write the difference between the Lagrange interpolation function and Hermitian interpolation functions ? Write the cases where these interpolation functions are used. Derive the interpolation function of a master linear triangular element(**Fig.4**) using Lagrange basis polynomial in the natural co-ordinate system. [5]

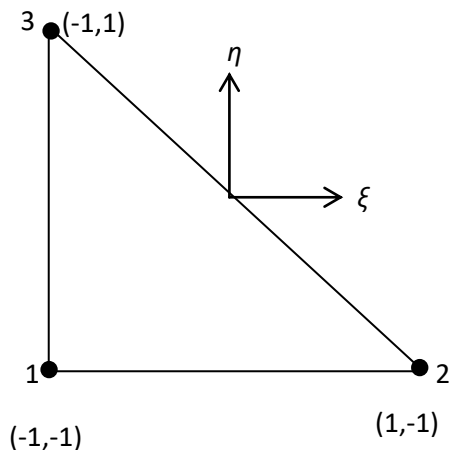


Fig.4 Master linear triangular element