BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Numerical Methods for Chemical Engineers (CHE F242) - Mid Semester Test

Date - 13/03/2023 **Closed Book** **Maximum Marks - 90**

1. A radioactive substance decays at a rate proportional to its concentration - that is:

$$\frac{dc}{dt} = -kc$$

Where c is the concentration of the radioactive substance measured in becquerel (Bq)/liter (l) and k is the decay constant (unit: day^{-1})
The initial concentration at time t = 0 is 100 Bq/l.
a) Determine the concentration after 0.6 days by analytical integration
b) Approximate the derivative term by forward finite divided difference operator and using a step size of $\Delta t = 0.3$, determine the concentration after 0.6 days.
c) Approximate the derivative term by forward finite divided difference operator and using a step size of $\Delta t = 0.2$, determine the concentration after 0.6 days.
d) Determine the true percentage relative errors in each of the above approximations.

[3+6+9+1]

2. The equation $x^2 = 5$ is to be solved using fixed point iteration method. Consider two alternative of this equation: $x = (x+5)/(x+1)$ and $x = (3x^2-5)/2x$. In the vicinity of the true value $x = \sqrt{5}$, which of the two alternatives will result in divergence from the solution and why? **[7]**

3. Explain two scenarios when the Newton-Raphson technique may diverge from the root due to an unfortunate selection of the initial guess value. By using Newton-Raphson technique, determine the root of $x^4 - x - 10 = 0$ which is near to x = 2 correct to three places of decimal. **[6+6]**

4. What are the pitfalls of naive Gauss elimination technique for solving system of linear algebraic equations? How does partial pivoting help in improving the naive Gauss elimination algorithm? Solve the following system of equations using Gauss-Siedel technique:

$$2x + 20y - 2z = -44$$
$$-2x + 3y + 10z = 22$$
$$10x + 2y + z = 9$$

Start with initial guess values of x = y = z = 0 and obtain the solution correct up to two significant figures. **[3+2+10]**

5. Explain 1-norm, ∞-norm and Frobenius norm for a matrix of dimension m×n. Determine the condition number of the 2×2 Hilbert matrix using row-sum norm (The Hilbert matrix is the square matrix given by $a_{ij} = \frac{1}{i+j-1}$. **[6+6]**

6. A company produces two types of products using two raw materials M_1 & M_2. The following table provides the basic data of the problem

	Tons of raw materials per ton of		Maximum daily availability
	Product 1	Product 2	
Raw material, M_1	6	4	24
Raw material, M_2	1	2	6
Profit per ton	5	4	

Using simplex method, determine the quantities of product 1 & product 2 to be produced per day in order to maximize profit and determine the maximum profit. **[25]**