Birla Institute of Technology and Science, Pilani

Semester I   Session: 2017-2018

CHE F414   Transport Phenomena

Mid-semester Test (Close Book)

Date: 12/10/2017   Maximum Marks: 25
Duration: 90 minutes   Weightage: 25 %

Q 1   **[4 x 1.5 = 6]**

(i) Explain the boundary conditions at
   a. Solid-fluid interfacial plane of constant z
   b. Liquid-liquid interfacial plane of constant x
   c. Liquid-gas interfacial plane of constant y

(ii) Explain Reynolds decomposition and turbulent momentum flux tensor

(iii) Consider steady state heat conduction (without any heat generation) for three different geometries (slab, cylinder and sphere). The temperature difference is applied in any one direction at a time. List the geometry and direction with corresponding linear/nonlinear temperature profile.

(iv) A solid sphere of radius *R* is rotating slowly (creeping flow) at a constant angular velocity Ω in a large body of quiescent fluid. The velocity v_ϕ is given by

$$v_\phi = \Omega R \left(\frac{R}{r}\right)^2 \sin\theta$$

Derive the formula of Torque required to maintain the rotation of the sphere.

Q 2   **[4]**

One half of a long cylindrical solid of radius *R* is exposed to the radiation heat flux from the sun while on the other half the radiation heat flux received is negligible (Fig. Q2). The rod is also losing heat to the surrounding (which is at T_∞) from its entire periphery by convection (neglecting radiation loss). Develop the steady state equation for the system and propose appropriate boundary conditions.

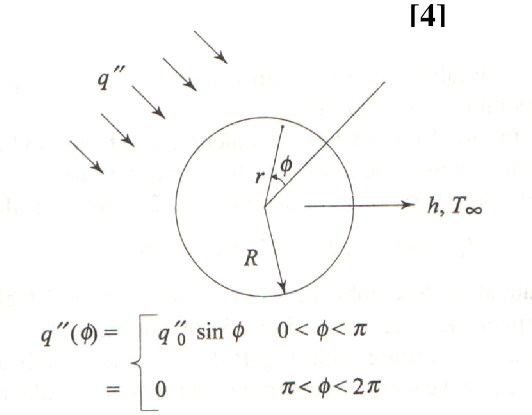


Fig. Q2

Q 3   **[5]**

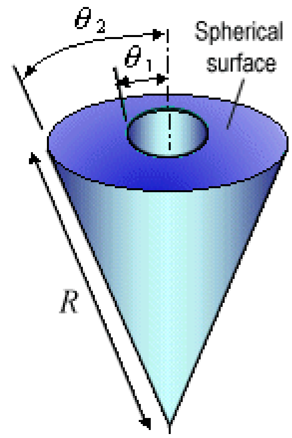
Two large flat porous horizontal plates are separated by a relatively small distance *L*. The upper plate at *y* = *L* is at temperature T_L, and the lower one at *y* = 0 is to be maintained at a lower temperature T_0. To reduce the amount of heat that must be removed from the lower plate, an ideal gas at T_0 is blown upward through both plates at a steady rate. Develop an expression for the temperature distribution and the amount of heat q_0 that must be removed from the cold plate per unit area as a function of the fluid properties and gas flow rate considering negligible viscous heat dissipation. Take constant thermal conductivity.

Q 4 **[5]**

A solid is formed from the conical section of a sphere of radius R as shown in the figure. The spherical surface at $r = R$ is insulated, while the two conical surfaces at $\theta = \theta_1$ and $\theta = \theta_2$ are held at temperatures T_1 and T_2, respectively. The thermal conductivity k of the solid material may be assumed constant.

a) Establish an expression for the temperature $T(\theta)$ in the solid object at steady state.

b) Find the total rate of heat flow across each of the conical surfaces.



Q 5 **[5]**

Liquid is present in the annular space between two vertical concentric cylinders of radius R_1 and R_2 ($R_2 > R_1$) that are rotating in opposite directions with the angular velocities of magnitude of Ω_1 and Ω_2. We would like to place a thin circular cylinder of negligible thickness and radius R_3 ($R_2 > R_3 > R_1$) concentric and in between with the two cylinders. Develop the relation to find the value of R_3 when no external torque would be required to hold this middle cylinder stationary. What is the value of R_3, if $R_2 = 2\,R_1$ & angular velocity of both cylinders are same?

§B.4 THE EQUATION OF CONTINUITY[a]

$$[\partial\rho/\partial t + (\nabla \cdot \rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \quad \text{(B.4-1)}$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \quad \text{(B.4-2)}$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi) = 0 \quad \text{(B.4-3)}$$

[a] When the fluid is assumed to have constant mass density ρ, the equation simplifies to $(\nabla \cdot \mathbf{v}) = 0$.

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \quad \text{(B.6-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \quad \text{(B.6-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-3)}$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-6)}$$

§B.2 FOURIER'S LAW OF HEAT CONDUCTION[a]

$$[\mathbf{q} = -k\nabla T]$$

Spherical coordinates (r, θ, ϕ):

$$q_r = -k\frac{\partial T}{\partial r} \quad \text{(B.2-7)}$$

$$q_\theta = -k\frac{1}{r}\frac{\partial T}{\partial\theta} \quad \text{(B.2-8)}$$

$$q_\phi = -k\frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi} \quad \text{(B.2-9)}$$

§B.7 THE DISSIPATION FUNCTION Φ_v FOR NEWTONIAN FLUIDS (SEE EQ. 3.3-3)

Cartesian coordinates (x, y, z):

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2 \qquad \text{(B.7-1)}$$

Cylindrical coordinates (r, θ, z):

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]^2 + \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2$$

$$- \frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right]^2 \qquad \text{(B.7-2)}$$

Spherical coordinates (r, θ, φ):

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta \cot\theta}{r}\right)^2\right]$$

$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2$$

$$- \frac{2}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2 \qquad \text{(B.7-3)}$$

§B.9 THE EQUATION OF ENERGY FOR PURE NEWTONIAN FLUIDS WITH CONSTANT[a] ρ AND k

$$[\rho \hat{C}_p DT/Dt = k\nabla^2 T + \mu\Phi_v]$$

Cartesian coordinates (x, y, z):

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \mu\Phi_v \qquad \text{(B.9-1)}^b$$

Cylindrical coordinates (r, θ, z):

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \mu\Phi_v \qquad \text{(B.9-2)}^b$$

Spherical coordinates (r, θ, φ):

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + \mu\Phi_v \qquad \text{(B.9-3)}^b$$

[a] This form of the energy equation is also valid under the less stringent assumptions k = constant and $(\partial \ln \rho/\partial \ln T)_p Dp/Dt = 0$. The assumption ρ = constant is given in the table heading because it is the assumption more often made.

[b] The function Φ_v is given in §B.7. The term $\mu\Phi_v$ is usually negligible, except in systems with large velocity gradients.