Molecular and Statistical Thermodynamics (CHE F415)

BITS Pilani, K.K. Birla Goa Campus

Mid-Semester Examination, 2022-23

Total Marks: 30 **Time Duration: 1.5 hr**

Problem 1: The probability distribution is given by,

$$f_x(x)=\left\{ cx^{\frac{n_1}{2}-1}\left(1+\frac{n_1}{n_2}x\right)^{-\frac{(n_1+n_2)}{2}} \right\} \text{ if } x \in R_x$$

$$f_x(x)=\{0\} \text{ if } x \notin R_x$$

$$\text{where, } c=\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}} \frac{1}{B\left(\frac{n_1}{2},\frac{n_2}{2}\right)} \quad , R_x = [0, \infty)$$

1) Find the mean value of the distribution. **5 Marks**

2) Find the variance of the distribution. **5 Marks**

Given Data:

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \int_0^\infty t^{x-1}(1+t)^{-x-y}dt$$

$$\Gamma(x+1) = x\Gamma(x)$$

$$\Gamma(x+1) = \int_0^\infty t^x e^{-t}dt$$

Problem 2: Derive the expression for enthalpy and specific heat capacity at constant pressure in terms of partition function. **5+5 = 10 Marks**

Given Data:

$$Q = \sum_j e^{-\beta E_j(N,V)}, \beta = \frac{1}{k_B T}$$

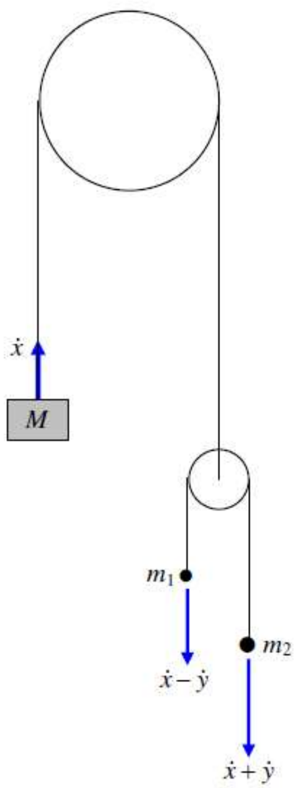
$$p_j(N,V,T) = \frac{{}^{-\beta E_j(N,V)}}{Q}$$

$$\langle E \rangle = -\left[\frac{\partial \ln Q(N,V,\beta)}{\partial \beta}\right]_{N,V} = k_B T^2 \frac{\partial}{\partial T}[\ln Q]$$

P.T.O.

Problem 3: The upper pulley is fixed in position. Both pulleys rotate freely without friction about their axles. Both pulleys are "light" in the sense that their rotational inertias are small and their rotation contributes negligibly to the kinetic energy of the system. The rims of the pulleys are rough, and the ropes do not slip on the pulleys. The gravitational acceleration is g. The mass M moves upwards at a rate $\dot{x}$ with respect to the upper, fixed, pulley, and the smaller pulley moves downwards at the same rate. The mass m_1 moves upwards at a rate $\dot{y}$ with respect to the small pulley, and consequently its speed in laboratory space is $\dot{x}-\dot{y}$. The speed of the mass m_2 is therefore $\dot{x}+\dot{y}$ in laboratory space. Find $\ddot{x}$ and $\ddot{y}$ in terms of g using Lagrangian Equation of Motion.

5 Marks



Given Data:

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \left(\frac{\partial L}{\partial q_k}\right) = 0$$

Problem 4: Derive Maxwell distribution in terms of Energy Distribution for spherical coordinates.

5 Marks

Given Data:

$$N(u)du = 4\pi N_0 \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} e^{\frac{-mu^2}{2k_BT}} u^2 du$$