Birla Institute of Technology and Science, Pilani

Semester I Session: 2016-2017

CHE G523 Mathematical methods in chemical engineering

Comprehensive Examination Test (Close Book + Open Book)

Date: 13/12/2016 Maximum Marks: 35

Duration: 180 minutes Weightage: 35 %

Close Book (Time:2hr)

Q 1 **[2 X 3 = 6]**

(a) Explain the difference between ill posed problem and well posed with an example.

(b) Apply Von-Neumann stability analysis to the pure convection unsteady state equation (1 D) to prove that FTCS (forward in time and central in space) scheme is unconditionally unstable.

(c) Explain in brief the procedure to solve the 2 D incompressible flow momemntum transfer equations with unknown pressure distribution.

Q 2 **[6]**

Generally, at the convective boundary condition, the conductive heat flux (at the boundary) is compared with convective heat flux for steady state heat transfer. Develop the finite difference equation for finding the value of temperature at boundary considering the heat capacity for the boundary (unsteady state). Also find the stability criterion for the explicit method.

Q 3 **[6]**

A fluid with variable density *ρ(T)* and constant viscosity μ is located between two vertical walls a distane L apart. The heated wall at $z = 0$ is maintained at temperature T_2 and cooled wall at $z = L$ is maintained at temperature T_1. The governing equations are given below.

$$\mu\frac{\partial^2 v_z}{\partial y^2} = \bar{\rho} g - \bar{\rho} g \bar{\beta}\left(T - \bar{T}\right); \ \bar{T} = \frac{1}{2}\left(T_1 + T_2\right) \qquad \text{and} \qquad k\frac{\partial^2 T}{\partial y^2} + \mu\frac{\partial}{\partial y}\left(v_z \frac{\partial v_z}{\partial y}\right) = 0$$

Using finite difference method, propose an algorithm to get temperature and velocity profiles between two plates.

Q 4 **[5]**

Consider an electric wire of circular cross section with radius R and electrical conductivity k_e, ohm^{-1} cm^{-1}. Through this wire there is an electric current with current density I amp/cm^2. The transmission of an electric current is an irreversible process, and some electrical energy is converted into heat (thermal energy). The rate of heat production per unit volume is given by the expression, $S_e = \frac{I^2}{k_e}$. The surface of the wire is maintained at temperatue T_0. Develop the non homogeneous ODE-BVP system with appropriate boundary conditions.

Using orthogonal collocation method (N+2 = 4), find the temperature at the centre of a copper wire having 2 mm radius, 5 m length and drawing 32 amp current. Consider surface temperature of 20 ˚C, electrical conductivity of 5.1 x 10^5 ohm^{-1} cm^{-1} and thermal conductivity of 400 W/m K.

MATRICES FOR THE ORTHOGONAL COLLOCATION TECHNIQUE (NON-SYMMETRIC)

N+2 = 4

X=

0
0.2113
0.7887
1

A=

-7.001	8.196	-2.196	1.000
-2.733	1.733	1.732	-0.732
0.732	-1.732	-1.733	2.733
-1.000	2.196	-8.196	7.001

B=

24.002	-37.177	25.176	-12.001
16.395	-24.002	12.001	-4.394
-4.394	12.001	-24.002	16.395
-12.001	25.176	-37.177	24.002

Open Book (Time: 1hr)

Q 1 **[8]**

As stell ball (20 cm diameter) initially at an ambient condition (30 deg C). Suddenly, the bottom hemisphere of the ball is buried into the ice brick (0 deg C). Whereas the top hemispehere is still exposed ot the atmosphere. Using the implicit finite difference method, convert the differential equationin a set of algebraic equations and propose the algorithm to solve them.

Equation of heat conduction in spherical coordinate:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}=\alpha\frac{\partial T}{\partial t}$$

Q 2 **[4]**

Solve the $\dot{x}=Ax+b$ using eigen value method.

$$A=\begin{bmatrix}5 & 8\\ -6 & -9\end{bmatrix};\qquad B=\begin{bmatrix}1\\ 1\end{bmatrix};\qquad x^0=\begin{bmatrix}0\\ 0\end{bmatrix}$$