Birla Institute of Technology and Science, Pilani

Semester I Session: 2017-2018

CHE G523 Mathematical methods in chemical engineering

Comprehensive Examination Test (Close Book + Open Book)

Date: 4/12/2017 Maximum Marks: 35

Duration: 180 minutes Weightage: 35 %

Close Book (Time:2hr)

Q 1 **[2 x 6 = 12]**

The air is blowing at 0.95 mm/s and between two parallel plates of 100 mm length at 300 K and leaving at 310 K. Consider this system as 1D steady state convection diffusion problem of heat transfer. The thermal diffusivity of air is 19 mm^2/s.

a) Develop the governing equation for this system to find the temperature profile and identify suitable boundary conditions. Introduce the Peclet Number and non dimensinoalise the equation.
b) Discretise the governing equation with central finite difference technique with grid spacing of 33.33 mm, and find the temperature profile.
c) Discretise the governing equation with first order upwind finite difference technique with grid spacing of 33.33 mm, and find the temperature profile.
d) Compare the results of (b) and (c) with analytical results and comment.
e) What should be minimum grid spacing to achieve computationally stable results?
f) What should be minimum grid spacing to achieve accurate and computationally stable results?

Q 2 **[6]**

Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in the domain $0 < x < 5$.

At $t = 0$, $u = 20$ for all x

for $t > 0$, at $x = 0$, $u = 0$

at $x = 5$, $u = 100$

Use explicit method and find maximum time step to achieve stable results with $\Delta x = 1$. Find the u profile at time t = 1. What would be profile shape after long time?

Q 3 **[1 +1+2+2 = 6]**

Consider the case of steady state heat conduction in a long square slab (2L x 2L) in which heat is generated ata a uniform rate of q W/m^3. The problem can be assumed to be of two dimensional one as the dimension of the slab is much longer in the direction normal to the cross sectional plane. All four sides are maintained at $T = T_\infty$, the temperature of the surrounding fluid, assuming the large heat transfer coefficient.

a) By applying the symmetry, reduce the domain and write governing energy equation with appropriate boundary conditions.
b) Non dimensinoalise the eqations and boundary conditions by introducing suitable nondimensinal variable
c) Discretize the equations using appropriate finite difference scheme anddevelop the generalized discretized equations for various grid points.
d) Propose an algorithm to solve the equations using line by line method.

Open Book (Time: 1hr)

Q 1 **[2+5+1+3 = 11]**

For a spherical geometry with $v_\phi = 0$ and no ϕ dependence, the equations of stream function can be given by following set of equations.

$$v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}, \quad v_\theta = +\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$$

$$\frac{\partial}{\partial t}(E^2\psi) + \frac{1}{r^2\sin\theta}\frac{\partial(\psi, E^2\psi)}{\partial(r,\theta)} - \frac{2E^2\psi}{r^2\sin^2\theta}\left(\frac{\partial\psi}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial\psi}{\partial\theta}\sin\theta\right) = \nu E^4\psi$$

Where

$$E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

$$E_s^4\psi_s = \frac{\partial^2}{\partial r^2}\left[\frac{\partial^2\psi_s}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi_s}{\partial\theta}\right)\right] + + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\partial^2\psi_s}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi_s}{\partial\theta}\right)\right)\right] = 0\,.$$

For a system having a stationary sphere of radius *R*, the Newtonian fluid is flowing around it at Re << 1. The fluid is approaching the shere in the positive z-direction as shown in Fig Q1. Assume that thre is no velocity variation beyond 2*R* radius.

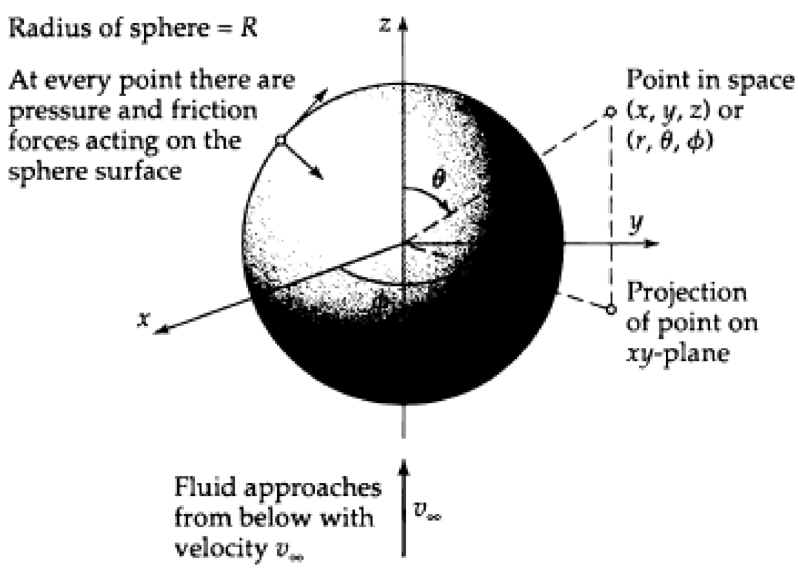

Fig. 2.6-1 Sphere of radius *R* around which a fluid is flowing. The coordinates *r*, *θ*, and *ϕ* are shown. For more information on spherical coordinates, see Fig. A.8-2.

a) Obtain the governing equation and boundary conditions in terms of stream function.
b) Discretise the governing equation with appropriate finite difference technique using $\Delta\theta = \pi/4$ and $\Delta r = R/4$ anddevelop the generalized discretized equations for various grid points.
c) Identify the line of symmetry and minimum no of algebraic equations required to be solved to get the stream function values at all grid points.
d) Propose an algorithm/equations to find the velocity profile