Birla Institute of Technology and Science, Pilani

Semester I Session: 2017-2018

CHE G523 Mathematical methods in chemical engineering

Mid-semester Test (Closed Book)

Date: 10/10/2017 Maximum Marks: 25

Duration: 90 minutes Weightage: 25 %

Q 1 **[4+5+4 = 13]**

The tempereature variation in a rod of 2 m is given by $\frac{d^2T}{dx^2} + 0.01(20 - T) = 0$. The value of temperature at $x = 0$ is 40 ˚C and at 2 m, its value is 200 ˚C.

a) Apply appropriate finite difference method assuming Δx = 0.5 m and develop the set of algebraic equations and solve them.

b) Convert above equation into a set of first oder equations. Apply the shooting method and solve the set of ODE -IVP using eigne value method.

c) Apply orthogonal collocation method for three internal collocation points and find the temperature profile.

Q 2 **[8]**

The half solid cylinder of 10 cm diameter has k = 20 W/m K and its top surface is exposed to the convection environment at 20 ˚C with h = 50 W/m^2 K. The lower bottom surface is maintained at 300 ˚C. Using finite difference method by considering Δr of 2.5 cm and $\Delta\theta$ of 45˚, find out the temperature profile. Also consider the minimum number of nodal points based on physical symmetry in temperature profile.

Energy Equation for cylindrical coordinates:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

Q 3 **[2+2 = 4]**

a) Generate an orthonormal set from the linearly independent set $(2,0,1)^t$, $(2,1,3)^t$, $(4,1,2)^t$ in R^3 using Gram-schmidt Orthonormalisation process.

b) Using the following symmetric matrix, show that eigen vectors are orthogonal to one another.

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

MATRICES FOR THE ORTHOGONAL COLLOCATION TECHNIQUE (NON-SYMMETRIC)

For N+2 = 3

X=

0
0.5
1

A=

-3	4	-1
-1	0	1
1	-4	3

B=

4	-8	4
4	-8	4
4	-8	4

N+2 = 5

X=

0
0.1127
0.5
0.8873
1

A=

-13.000	14.788	-2.667	1.878	-1.000
-5.324	3.873	2.066	-1.291	0.676
1.500	-3.228	0.000	3.228	-1.500
-0.676	1.291	-2.066	-3.873	5.324
1.000	-1.878	2.667	-14.788	13.000

B=

84.001	-122.064	58.666	-44.604	24.000
53.239	-73.334	26.667	-13.334	6.762
-6.000	16.667	-21.333	16.667	-6.000
6.762	-13.334	26.667	-73.334	53.239
24.000	-44.604	58.666	-122.064	84.001