# BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI 

First Semester 2022-2023
CHE G523 Mathematical Methods in Chemical Engineering Comprehensive Examination
Date: 24.12.2022, 2-5 PM
Duration: 180 Min.
Total Marks: 35

## CLOSED BOOK (10 Marks)

Q. 1 Using the symmetric matrix $A=\left[\begin{array}{ccc}1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1\end{array}\right]$, Compute the Eigen values.
[2 Marks]
Q. 2 Write a MATLAB code to solve the ODE: $\frac{d^{2} T}{d x^{2}}+h\left(T_{a}-T\right)=0$ with boundary conditions: $T(0)=T_{1}$ and $T(L)=T_{2}$.
[2 Marks]
Q. 3 Consider the splitter network shown in the figure: $\boldsymbol{F}$ represents the mass flow rate. Formulate the system of vectorial form, and comment on independency and how to obtain the solution.

[2 Marks]
Q. 4 Using the Finite Difference approach with explicit method, derive the recurring expression for the solution of the following PDE.

$$
\frac{\partial C}{\partial t}=D \frac{\partial^{2} C}{d x^{2}}
$$

at $t=0$ is $0 \mathrm{~mol} / \mathrm{cm}^{3}$ for all $x$.
for $t>0 \quad \mathrm{C}$ at $x=0$ is $5 \mathrm{~mol} / \mathrm{cm}^{3}$.
$x=10 \mathrm{~cm}$ is $\frac{\partial C}{\partial x}=0$.
Use a time step of 0.2 s , and 4 intervals for length coordinate. $D=2$.
[2 Marks]
Q. 5 A reversible reaction $A \leftrightarrow B$ occurs isothermally in a batch reactor. The evolution of concentrations is given by:

$$
\frac{d C_{A}}{d t}=-C_{A}+2 C_{B} \text { and } \frac{d C_{B}}{d t}=C_{A}-2 C_{B}
$$

The initial concentrations of $A$ and $B$ are $2 \mathrm{~mol} / \mathrm{cc}$ and $3 \mathrm{~mol} / \mathrm{cc}$, respectively. Using eigenvalue method, determine the equilibrium concentration of A and B .

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## OPEN BOOK (25 MARKS)

Q. 1 The reaction-diffusion equation for a catalyst slab in which a first-order reaction occurring is:

$$
\frac{d^{2} C}{d x^{2}}=\phi^{2} C
$$

Where $C$ and $x$ are dimensionless concentration and length variables, respectively. $\phi$ is the Thiele modulus, which can be taken as 0.5 .
The boundary conditions for the equation are: $C(0)=0$ and $C(1)=1$.
Solve the equation:
(A) Using the Orthogonal Collocation method, solve the governing equation for $\mathrm{N}=2$.
(B) Using the Finite Difference method, solve the governing equation taking three intervals.
(C) Using the Shooting method and Euler forward method with $h=1 / 3$

Tabulate $x$ vs $C$ for all three methods.

$$
[3 \times 4 \text { Marks }=12 \text { Marks }]
$$

Q. 2 Consider the Non-Dimensional unsteady convection-diffusion heat transfer equation

$$
\frac{\partial T}{\partial t}+\frac{\partial T}{\partial x}=\frac{1}{P e}\left[\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}\right]
$$

(A) Derive the finite difference upwind implicit formulation for the above PDE. Write the Gauss-Seidel formulation of the problem.
(B) Now assume steady state, we get convection-diffusion heat transfer equation as:

$$
\frac{\partial T}{\partial x}=\frac{1}{P e}\left[\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}\right]
$$

(i) Derive the recurring equation with FD method with central scheme.
(ii) Solve the above equation using the finite difference method with the central scheme and $\Delta x=1 / 3, \Delta y=1 / 2$ and boundary conditions as shown in the figure and peclet number is 1.5


$$
[4+9=13 \text { Marks }]
$$

## \#All The Best\#

