BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2023-2024
CHE G523 Mathematical Methods in Chemical Engineering
Mid Semester Exam (Closed Book)

Date: 09.10.2023, 4-5:30 PM **Duration: 90 Min.** **Total Marks: 25**

- State and justify any assumptions that you make. Nomenclature should be defined properly.

Q.1 **[4*2=8 Marks]**

a) Prove the theorem: "The eigenvector u^i of a real matrix A corresponding to λ_i is orthogonal to the complex conjugate of every eigenvector v^j of A^t corresponding to an eigenvalue λ_j distinct from λ_i."

b) Integrate $\frac{dy}{dt} = -3y$ with initial value $y_0 = 1$ using $h = 0.025$ upto $t = 0.25$ using explicit Euler technique. Compare with the exact values.

c) Calculate the metrics $d_1(u,v)$, $d_2(u,v)$, and $d_\infty(u,v)$ for $u = (1 \quad 4 \quad 6)^t$ and $v = (1 \quad -2 \quad 0)^t$

d) Express vector $u = (8 \;\; 10 \;\; 12)^t$ in $\mathbf{R}^3$ as a linear combination of basis sets $u^1 = (1 \;\; 2 \;\; 3)^t$, $u^2 = (3 \;\; -3 \;\; 1)^t$ and $u^3 = (-11 \;\; -8 \;\; 9)^t$

Q.2 Using the data given below fit a third-degree polynomial using the Lagrangian interpolation formula.

x	1	3	4	6
y	1	5	15	85

The Lagrangian interpolation formula is given by $y(x) = \sum_{j=0}^{n}\left[\prod_{\substack{i=0 \\ i\neq j}}^{n} \frac{(x-x_i)}{(x_j-x_i)}\right] y_j$

[3 Marks]

Q.3 Consider linearly independent set $(2, 0, 1)^t$ $(2, 1, 3)^t$ $(4, 1, 2)^t$ in $\mathbf{R}^3$. Generate an orthonormal set using the Gram-Schmidt Orthonormalisation process. **[3 Marks]**

Q.4 Consider the following ODE-IVP

$$\frac{dy_1}{dt} = 9y_1 + 24y_2 + 5\cos t - \frac{1}{3}\sin t$$

$$\frac{dy_2}{dt} = -24y_1 - 51y_2 - 9\cos t + \frac{1}{3}\sin t$$

With initial $y_1(0) = 4/3$ and $y_2(0) = 2/3$. Integrate the ODE using the Runge-Kutta algorithm with h=0.05 for 1 step. **[3 Marks]**

Fourth order Runge-Kutta algorithm:

$$y_{n+1} = y_n + h\left[\frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4\right]$$

$k_1 = f[t_n, y_n]$ $\quad k_2 = f\left[t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right]$ $\quad k_3 = f\left[t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right]$ $\quad k_4 = f[t_n + h, y_n + hk_3]$

Q.5 Determine the heights h_1, and h_2 in the two-tank network using the eigenvalue method for steady state and transient response. The equations are given by:

$$\frac{dh_1}{dt} = \frac{F_i}{A_1} - \left(\frac{h_1 - h_2}{R_1A_1}\right)$$

$$\frac{dh_2}{dt} = \left(\frac{h_1 - h_2}{R_1A_2}\right) - \left(\frac{h_2}{R_2A_2}\right)$$

Where $F_i/A_1 = 1$, $R_1A_1 = 0.5$, $R_2A_2 = 0.5$, $R_1A_2 = 1$ **[8 Marks]**

~ALL THE BEST~