BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2023-2024
CHE G523 Mathematical Methods in Chemical Engineering
Comprehensive Examination

Date: 07.12.2023, 2-5 PM | **Duration: 180 Min.** | **Total Marks: 35**

OPEN BOOK (35 Marks)

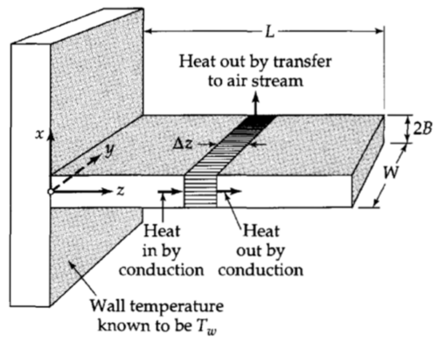
Note: Clearly label the answer. Start the answer to each question on a separate sheet.

Q.1 A heat conducts through a rectangular fin ($B<<L$ and $B<<W$) is shown in the figure. The wall temperature is T_w and the ambient temperature is T_a. The heat transfer can be described by the equation:

$$\frac{d^2T}{dz^2} = \frac{h}{kB}(T - T_a)$$

The boundary conditions for the problem are:

$$z = 0 \qquad T = T_W$$
$$z = 1\,cm \qquad \frac{dT}{dz} = 0$$



The ambient temperature is 30ºC and the temperature of the wall is 100ºC. The heat transfer coefficient (h) is 5 W/(m^2.ºC). The thermal conductivity (k) of metal is 400 W/(m.ºC). The thickness (2B) is 4 mm.

(a) Solve using orthogonal collocation method, with N = 2, find the temperature distribution in the fin.
(b) Solve using the Shooting method and Euler forward method with $h = 1/3$

[5+3 = 8 Marks]

Q.2 A steady laminar flow in a circular tube of radius (R) is described by the equation,

$$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{dP}{dz}$$

The pressure gradient may be taken as constant, i.e., $\frac{dP}{dz} = \mathbf{C}$. The boundary conditions for the problem are

$$at\ r = R\,, \quad v = 0 \quad and \quad at\ r = 0\,, \quad \frac{dv}{dr} = 0$$

Using finite difference method, and 4 grid points (including boundaries), find the velocity distribution. The viscosity μ can be taken as 1 and **C** as 10.

[8 Marks]

Q.3 A reversible reaction $A \leftrightarrow B \leftrightarrow C$ occurs isothermally in a batch reactor. The forward and backward rate constants for $A \leftrightarrow B$ are 1 s^{-1} and 2 s^{-1}. The forward and backward rate constants for $B \leftrightarrow C$ are 2 s^{-1} and 1 s^{-1}. Assume first order elementary reactions.
(a) Write the governing equations for the evolution of concentrations.
(b) The initial concentrations of A, B, and C are 2 mol/cc, 3 mol/cc, and 4 mol/cc, respectively. Using eigenvalue method, determine the equilibrium concentration of A and B.

[6 Marks]

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2023-2024
CHE G523 Mathematical Methods in Chemical Engineering
Comprehensive Examination

Date: 07.12.2023, 2-5 PM **Duration: 180 Min.** **Total Marks: 35**

Q.4 Consider an infinite slab of thickness L. Initially, the body is at a uniform temperature, T_i, Suddenly, one side is exposed to a very hot environment at temperature T_∞, the other side being insulated.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{dx^2}$$

at $t = 0$ is T_i for all x.

for $t > 0$ T at $x = 0$, $\frac{dT}{dx} = \sigma\epsilon(T_\infty^4 - T^4)$ is the radiation boundary condition.

x = L , $\frac{dT}{dx} = 0$

Where σ is the Stefan Boltzmann constant, and ϵ is the emissivity of the slab.

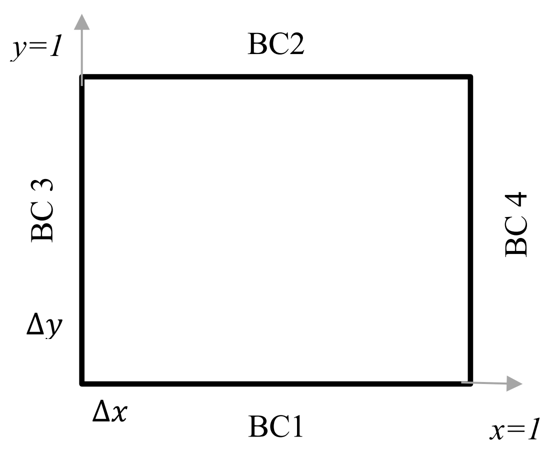
Develop the finite difference method using explicit scheme to find the temperature distribution in the slab as a function of time.

[5 Marks]

Q.5 Solve the equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + 1 = 0$ for a square plate and with given boundary conditions using Finite difference method.

(a) Develop the method and solve using Liebmann's method (Gauss-Siedal method). Divide into four equal intervals in x, and three equal intervals y.
(b) At each grid point, find the heat flux and the direction assuming thermal conductivity as 25 W/(m.ºC).

BC1=Insulated, BC2=100°C, BC3 = Insulated, and BC4=50°C



[8 Marks]

#All The Best#