BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Chemical Process Optimization (CHE G558), Mid Semester Examination

Date – 12/10/2023 Open Book Maximum Marks - 30

1. A manufacturing firm produces two products, A and B, using two limited resources. The maximum amounts of resources 1 and 2 available per day are 1000 and 250 units, respectively. The production of 1 unit of product A requires 1 unit of resource 1 and 0.2 unit of resource 2, and the production of 1 unit of product B requires 0.5 unit of resource 1 and 0.5 unit of resource 2. The unit costs of resources 1 and 2 are given by the relations (0.375 − 0.00005u1) and (0.75 − 0.0001u2), respectively, where u_i denotes the number of units of resource i used (i = 1, 2). The selling prices per unit of products A and B, p_A and p_B, are given by
   $p_A = 2.00 - 0.0005x_A - 0.00015x_B$ and $p_B = 3.50 - 0.0002x_A - 0.0015x_B$
   x_A and x_B indicate, respectively, the number of units of products A and B sold.
   Formulate the problem of maximizing the profit assuming that the firm can sell all the units it manufactures. [**4**]
2. Growth rate of bacteria k (d^{-1}), as a function of oxygen concentration c (mg/L) can be modeled by the following equation: $= \frac{k_{max}c^2}{c_s+c^2}$. where c_s and k_{max} are parameters. Use the data below to estimate the parameters (Apply linear regression technique) [**5**]

c	0.5	0.8	1.5	2.5	4
k	1.1	2.4	5.3	7.6	8.9

3. Determine the extreme (stationary) points of the following function.
   $$f(x_1,x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$
   Classify all the extreme points as minimum, maximum or saddle point. [**7**]
4. Minimize the function $f(x) = 0.65 - \left[{}^{0.75}/_{(1+x^2)}\right] - 0.65x\tan^{-1}({}^{1}/_{x})$
   Using golden section search. Start with an interval [0, 3] and perform three iterations [**5**]
5. Consider the minimization of the function $f(x_1,x_2) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - 2x_2$
   If $S_1 = \begin{Bmatrix}1\\2\end{Bmatrix}$ denotes a search direction, find a direction S_2 that is conjugate to the direction S_1.
   [**5**]
6. It is required to minimize the function $(x_1,x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$. Starting from point $X_1 = \begin{Bmatrix}0\\0\end{Bmatrix}$, determine X_2 using steepest-descent method. [**4**]