# Birla Institute of Technology \& Science, Pilani, Rajasthan 333031 <br> Second Semester 2022-2023 <br> Comprehensive Examination <br> Course Name: Electron Correlation in Atoms and Molecules <br> Course Code: CHEM F413 <br> Date: 15-05-2023, Time: 1 Hrs 45 minutes; Max. Marks: 25 <br> (CLOSSED BOOK) <br> Answer all questions. Answers should be brief, relevant and to the point: 

1. (a) A system in a particular state is represented by the wave function $\left|\Psi_{1} \Psi_{2}\right\rangle$ and in another state represented by the wave function $\left|\psi_{1} \bar{\psi}_{2}\right\rangle$. The corresponding energies of the two states are represented by $E(\uparrow \uparrow)$ and $E(\uparrow \downarrow)$, respectively. Prove which one is lower. What does it signify?
$[1.5+1.5+1=4]$
(b) One of the diagonal elements in the full CI matrix of minimal basis $\mathrm{H}_{2}$ model is given as $\langle 3| h|3\rangle+\langle 4| h|4\rangle+\langle 34 \mid 34\rangle-\langle 34 \mid 43\rangle$ (in spin orbital form and physicist's notation). Derive the corresponding spatial orbital form in chemist's notation. Here, the notation convention is, $1 \equiv \chi_{1}(x) \equiv \psi_{1}(r) \alpha(w), \quad 2 \equiv \chi_{2}(x) \equiv \psi_{1}(r) \beta(w)$, $3 \equiv \chi_{3}(x) \equiv \psi_{2}(r) \alpha(w)$ and $4 \equiv \chi_{4}(x) \equiv \psi_{2}(r) \beta(w)$
(c) Prove that the Coulomb operator is invariant to Unitary transformation.
2. (a) If the net dipole moment ( $\vec{\mu}$ ) of a molecule is represented by $\vec{\mu}=\sum_{i} q_{i} \vec{r}_{i}+\sum_{A} Z_{A} R_{A}$ (where the symbols have their usual meaning) then what are the two terms in the right hand side of the above equation represent here? Write the expression of the dipole moment in density matrix form explaining all the terms.
[ $0.5+1.5=2$ ]
(b) Draw the pictorial form of the Slater determinant of an unrestricted open-shell system. How do you express $\rho_{\mu \nu}^{\alpha}$ (i.e. the $\mu \nu^{\text {th }}$ element of the charge-density bond-order matrix corresponding to $\alpha$-electron) and $\rho_{\mu \nu}^{\beta}$ (i.e. the $\mu \nu{ }^{\text {th }}$ element of the chargedensity bond-order matrix corresponding to $\beta$-electron)
$[1+1+1=3]$
3. (a) Starting from the intermediate normalized form of CI wave function show that,

$$
E_{c o r r}=\sum_{\substack{c<d \\ t<u}}^{N} C_{c d}^{t u}\left\langle\Psi_{0}\right| H\left|\Psi_{c d}^{t u}\right\rangle
$$

where the symbols have their usual meaning.
(b) Comment on the variational and size consistency properties of Configuration Interaction (CI) and Möller - Plesset perturbation theory (MPPT) based methods. [3]

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Answer all questions. Answers should be brief, relevant and to the point:

1. (a) If for $\mathrm{H}_{2} \mathrm{O}$ molecule $\sum_{\mu \in H_{1}}(P S)_{\mu \mu}=0.76442$ (where $\mu$ 's are the basis functions, $P$ is the 'charge-density bond-order' matrix, $S$ is the overlap matrix), then what will be the values of $\sum_{\mu \in H_{2}}(P S)_{\mu \mu}$ and $\sum_{\mu \in O}(P S)_{\mu \mu}$ ? Here, $H_{1}, H_{2}$ and $O$ represent the two hydrogen atoms and the oxygen atom respectively.
(b) If the net dipole moment ( $\vec{\mu}$ ) of a molecule is represented by $\vec{\mu}=\sum_{i} q_{i} \vec{r}_{i}+\sum_{A} Z_{A} R_{A}$ (where the first and second terms are the electronic and nuclear contributions, respectively, and the symbols have usual meaning) then what will be the dipole moment of $\mathrm{CO}_{2}$ molecule? Explain (in not more than five sentences).
$[1+2=3]$
(c) The total energy ( $E_{T o t}$ ) of a chemical system is represented by
$E_{T o t}=E_{0}+\sum_{A} \sum_{B>A} \frac{Z_{A} Z_{B}}{R_{A B}}$, where $E_{0}$ is the 'electronic energy' and $\sum_{A} \sum_{B>A} \frac{Z_{A}^{Z} B}{R_{A B}}$ is the nucleus-nucleus repulsion energy. Which of the following statement is correct and why (answer within five sentences)?
(i) $E_{T o t}>E_{0}$
(ii) $E_{T o t}<E_{0}$
(d) What will be the total number of basis functions and primitive Gaussians for $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}$ molecule in $6-31 \mathrm{G}^{* *}$ basis set?
(e) Showing relative position of energy values of neutral, cationic and anionic systems explain why Koopmans' IP a somewhat acceptable approximation of actual IP whereas Koopmans' EA is not.
