# Birla Institute of Technology \& Science, Pilani, Rajasthan - 333031 <br> Mid-Semester Test, First Semester, AY: 2017-2018 <br> CHEM G553: Advanced Physical Chemistry 

Time: 90 minutes
Max. Marks: 90
There are six questions. Attempt all the questions. Pencil should not be used. Don't scribble on the question paper. Useful date: $\mathrm{h}=6.63 \times 10^{-34} \mathrm{~J} \mathrm{~s}$; Velocity of light $=2.998 \times 10^{8} \mathrm{~ms}^{-1}$; Mass of the electron $=9.1 \times 10^{-31} \mathrm{~kg}$, $1 \mathrm{amu}=1.66054 \times 10^{-27} \mathrm{~kg}$

1 (a) Show that a function $x^{k}$ is an eigen function of the operator, $\widehat{\boldsymbol{O}}=\left[\mathrm{a}+\mathrm{bx}\left(\frac{d}{d x}\right)\right]$, where k , a , and b are constants. What is the eigen value? Is the function $f(x)=x^{3}+3 x$ an eigen function of the given operator?
(b) In an X-ray photoelectron experiment, a photon of wavelength 150 pm ejects an electron from the inner shell of an atom and it emerges with a speed of $21.4 \times 10^{6} \mathrm{~m} / \mathrm{s}$. Calculate the binding energy of the electron in Joule.
(c) Which of the following expressions are acceptable wave functions? Briefly justify
(i) $\Psi=x^{2}+1$, where $x$ can have any value
(ii) $\Psi=1 /(4-x) ; 0 \leq x \leq 3$

2 (a) A particle of mass $m$ is confined to move in two dimension. The potential energy $\mathrm{V}=0$ for $0 \leq \mathrm{x}$ $\leq L_{1}, 0 \leq y \leq L_{2}$ and $V=\alpha$ elsewhere. Answer the following questions:
(i) Write down the form of the normalized wave function for the particle.
(ii) Write down the expression for energy of the particle.
(iii) Consider, $L_{1}=L$ and $L_{2}=2 L$. Show that there is a degeneracy between the states $|1,4\rangle$ and |2,2>.
(b) What is the magnitude of the momentum of a free particle having momentum eigen function, $\Psi$ $=e^{i 4 x}$ ?
3 (a) Determine the normalization constant, N , for the wave function $\Psi(\phi)$ for particle of mass m on a ring which is having the form, $\Psi(\phi)=\mathrm{Ne}^{\mathrm{im} \phi}$.
(b) Evaluate the magnitude of angular momentum associated with the particle of mass m as described in the previous question. Wave function of the particle is $\Psi(\phi)=\mathrm{Ne}^{\mathrm{im} \phi}$.
(c) Consider a diatomic molecule, ${ }^{1} \mathrm{H}^{35} \mathrm{Cl}$. The force constant of the bond is $\mathrm{k}_{\mathrm{f}}=516.3 \mathrm{~N} \mathrm{~m}^{-1}$. Determine the frequency of the oscillation for ${ }^{1} \mathrm{H}^{35} \mathrm{Cl}$. Determine the energy separation between the $v=0$ and $v=1$ states.
4 (a)


Orbital angular momentum quantum number of a rigid rotor, represented in the Figure, is ' $I$. Answer the following
(i) Magnitude of slant height, $\boldsymbol{p}$, of the total angular momentum vector 'L'.
(ii) Magnitude of the altitude, $\boldsymbol{q}$.
(iii) How many values are possible for $\boldsymbol{q}$.
(iv) Why the vector $\mathbf{L}$ lie on the surface of a cone? Explain in one or two sentences.
(b) Consider an electron in the $3 p_{x}$ orbital of H -atom. Answer the following questions:
(i) What would be outcome(s) of the $\widehat{L_{Z}}$ operator? Do mention the probability of each outcome(s).
(ii) What would be the value(s) of spin magnetic quantum number of the electron?
(c) Fluorescence emission spectra of molecules in the solution are in general independent of the excitation wavelength. Explain briefly.
(d) The molar absorption coefficient of solute at 540 nm is $286 \mathrm{dm}^{3} \mathrm{~mol}^{-1} \mathrm{~cm}^{-1}$. When light of that wavelength passes through a 6.5 mm cell containing a solution of the solute, 46.5 percent of light absorbed. What is the concentration of the solution?
5 (a) Normalize the molecular orbital, $\varphi=\psi_{A}+\lambda \psi_{B}$ in terms of the parameter $\lambda$ and the overlap integral S.
(b) Arrange the species $\mathrm{O}_{2}{ }^{+}, \mathrm{O}_{2}, \mathrm{O}_{2}^{-}$, and $\mathrm{O}_{2}{ }^{2-}$ in order of increasing bond order. Do mention the bond order of each species. What is the spin state of ground state O 2 molecule?
(c) The lowest observed microwave absorption frequency of ${ }^{12} \mathrm{C}^{16} \mathrm{O}$ [Reduced mass, $\mu=1.13850$ $\times 10^{-26} \mathrm{~kg}$ ] is 115271 MHz . Compute (a) bond distance in ${ }^{12} \mathrm{C}^{16} \mathrm{O}$ and (b) predict the next two lowest microwave absorption frequencies of ${ }^{12} \mathrm{C}^{16} \mathrm{O}$.
6 (a) Symmetric stretch vibration of $\mathrm{CO}_{2}$ is IR-inactive. Explain briefly.
Can you measure the frequency of the same mode using any spectroscopic method? Explain you answer.
(b) A transition is observed at $3185 \mathrm{~cm}^{-1}$ while recording vibrational spectrum of water. Other transitions, appeared at $3760 \mathrm{~cm}^{-1}$ (anti-symmetric stretch), $3650 \mathrm{~cm}^{-1}$ (symmetric stretch), and $1600 \mathrm{~cm}^{-1}$ (bending). Answer the following questions:
(i) Determine the zero-point vibration energy of water.
(ii) Assign the transition observed at $3185 \mathrm{~cm}^{-1}$.
(c) Calculate resonance frequency for proton in a 12.0 T magnetic field. $\gamma_{\mathrm{N}}$ is $2.6752 \times 108 \mathrm{~T}^{-1} \mathrm{~s}^{-1}$.
(d) Consider a nucleus with $\delta=1.00$ in a 500 MHz NMR spectrometer. Calculate the shift of the nucleus relative to the reference. What would be the shift in 100 MHz instrument?

