CS F211 - Data Structures and Algorithms - Comprehensive Exam (Closed Book)
$\qquad$ Name: $\qquad$ Signature:

## [Instructions:

- Answer each question only in the space provided. Binary Marking.
- Write your recheck requests only in the space provided below.
- Write neatly. Overwritten answers will not be considered for recheck.
- If you have any confusion, state your assumption and answer.

End of Instructions]
Recheck requests:

1. The data structure used for efficient Breadth First Traversal of a graph is $\qquad$ .1M
2. The data structure used for efficient Depth First Traversal of a graph is $\qquad$ 1M
3. Which graph traversal can be used to efficiently check if a graph is bipartite or not? Ans: 1M
4. A planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. Basically no edges cross each other. Which graph traversal can be used to efficiently check if a graph is a planar graph or not?

Ans:
5. Which graph traversal is most efficient for checking if a graph is having a cycle or not? Ans: $\qquad$ 1M
6. Which data structure can be used to efficiently implement Huffman Encoding Tries? Ans: $\qquad$ 1M
7. List two data structures that are used to implement Kruskal's algorithm efficiently. 2M
(i)
8. The minimum number of keys that can be stored in a B-tree with degree $\mathbf{t}$ and height $\mathbf{h}$ is $\qquad$ . And the maximum number of keys that can be stored in the above tree is $\qquad$ 2M
9. Suppose you want to implement the Search () function using binary search instead of linear search within each node of the above B-Tree. The time complexity of this Search() function is $\qquad$ 1M
10. What is the running time complexity of depth-first search, as a function of $|\mathrm{V}|$ and $|\mathrm{E}|$, if the input graph is represented by an adjacency matrix instead of an adjacency list?
11. The maximum difference between the depths of two leaves of a max heap is $\qquad$ 1M
12. Given a set of elements $\{1,2,3,4,5,6,7\}$. In what order should they be inserted into an AVL tree such that the tree doesn't have to undergo rotations? 3M

Ans:
13. Construct a Binary Tree with the given pre-order and in-order traversals as follows:

Pre-order: M, T, R, D, H, G, Q, Z, K, N, P. and In-order: R, T, H, D, M, Z, Q, K, G, N, P. Draw the binary tree.
(a) Draw the binary tree After constructing the tree (b) determine its post-order traversal. $\quad \mathbf{2 + 1}=\mathbf{3 M}$

Binary Tree
Post Order Traversal
14. A binary search tree contains $n$-distinct elements. What is the average case time complexity to select an element that is greater than the smallest element? Justify. $\mathbf{1 + 1 = 2 M}$
15. Which traversal method will you prefer to move from the smallest element to the highest element of this BST with the least cost? Justify your answer. 1+1=2M
$\square$
16. Write a single line statement in $C$ that verifies whether two vertices $V_{x}$ and $V_{y}$ belong to the same connected component of a graph G , using disjoint-set data structure? Assume necessary functions. 1M
17. (a) Which of the graph representations is suitable for representing sparse graphs? Justify. (b) And which is suitable for representing dense graphs? Justify. 4M
(a)
(b)
19. Given a graph as shown below:


Give a possible topological ordering of the above graph:
20. Let G be a graph with $\mathbf{n}$ vertices and $\mathbf{m}$ edges. What is the tightest upper bound on the running time on Depth First Search of G ? Assume that the graph is represented using adjacency matrix. Ans:
21. To implement Djikstra's algorithm on unweighted graphs so that the algorithm runs most efficiently, what is the choice of our data structure to implement it? Ans: 1M
22. Given a unweighted, undirected connected graph G. Devise a simple algorithm to compute shortest path from a node $S$ to all other nodes in $G$. Your algorithm should be most efficient in running time. Write your answer in one or two English sentences only. No need to give pseudo code. 3M

## [Instructions:

- Answer all the questions in the main answer sheet.
- Ensure the numbering of questions in the answer paper is correctly maintained.
- Sub-optimal solutions that don't adhere to the specified time complexities will not be credited.
- If you have any confusion, please state your assumptions and answer.


## End of Instructions]

1. Given an unsorted integer array nums. write an algorithm to find the smallest missing positive integer. Your algorithm should runs in $\mathbf{O}(\mathbf{n})$ time and use $\mathbf{O ( 1 )}$ extra space only. Write pseudo code only. $\mathbf{6 M}$
2. Consider a Binary Tree (BT) containing a set of $n$ keys ( $n>2$ ). Write an algorithm to verify whether the BT is a Binary Search Tree or not. Write pseudo code only.
3. Write an algorithm to verify whether a word/string is a palindrome (even-odd length both) or not, using a single stackbased operations (push and pop functions only) in most efficient way. Write pseudo code only.
4. Write an algorithm to find-out all the even numbers lesser than the $k$-th smallest element present in a BST containing even-odd numbers, with the best possible time complexity. Write pseudo code only.
5. Suppose, you are given an arithmetic expression to compute its final value. Suggest most efficient algorithm to model this problem and perform the computation most efficiently. Write pseudo code only.
6. Given an array $\mathbf{A}=[\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{I}, \mathrm{J}]$ that represents a binary min-heap containing 10 items. The key of each item is a distinct integer. State all possible items from the above array that could have the key with: (a) the smallest integer; (b) the third smallest integer; (c) the largest integer.
7. In a science-project exhibition contest, each participant is evaluated by the experts with a positive integer score within 100. Each participating project is assigned by a pair of positive integers ( $\boldsymbol{m}_{\boldsymbol{i}}, \boldsymbol{r}_{\mathbf{i}}$ ), where $\boldsymbol{m}_{\boldsymbol{i}}$ is the awarded score and $\boldsymbol{r}_{\boldsymbol{i}}$ is the participant's registration number. The selection committee wants to award identical trophies to the top $\mathbf{n}$ projects.
[Note: Write your answer within 5 clear sentences for each part. DON'T write pseudo code. End of Note]
a. Assume, $\mathbf{A}$ is an unsorted array of project pairs and a positive integer $\mathbf{n} \leq|\mathbf{A}|$.

Describe an algorithm with $\mathbf{O}(|\mathbf{A}|+\mathbf{n} \log |\mathbf{A}|)$ running time to return the registration numbers of $\mathbf{n}$ projects in $\mathbf{A}$ with the highest scores, breaking the ties (if any) arbitrarily.
b. Also, the committee decides to give encouraging certificates to each of those projects whose scores ( $\boldsymbol{m}_{\mathrm{i}}$ ) are strictly greater than a threshold score, $\boldsymbol{t}$. Given $\mathbf{A}$ with the ( $\mathbf{m}_{\mathbf{i}}, \mathbf{r}_{\mathbf{i}}$ ) pair of each participant, describe the most efficient algorithm stating the time and space complexity to return the registration numbers satisfying the above statement.
8. Given an array A of positive integers. Your task is to find the leaders in the array. An element of an array is a leader if it is greater than or equal to all the elements to its right side. The rightmost element is always a leader. Your algorithm should be most optimal in terms of auxiliary space used and overall run time complexity.
9. Mr. Robin is driving his new electric car from Pilani to Goa. He wants to take the shortest path, but his car can only drive $\boldsymbol{m}$ miles before requiring to charge. Fortunately, there are some electrical vehicle charging stations on his path, which instantaneously can charge the battery to full. The road map is given below as a weighted undirected graph $\mathbf{G}=(\mathbf{V}, \mathbf{E}, \mathbf{w})$ along with the subset $\mathbf{C} \subseteq \mathbf{V}$ of vertices that have charging stations. Each weight $\boldsymbol{w}(\boldsymbol{e})$ denotes the (positive) length of road $\boldsymbol{e}$. The aim is to find a shortest path from node $\mathbf{s} \in \mathbf{V}$ to node $\mathbf{d} \in \mathbf{V}$ that does not travel more than miles between charging stations. Assume that $\mathbf{s}, \mathbf{d} \in \mathbf{C}$. Charging stations are marked with car symbols within circles.

a. Find out the shortest path from Pilani to Goa in the following graph, if $m=\infty$. 3M
b. Find out the shortest path from Pilani to Goa in the following graph, if $m=101$ miles. 3M
c. Find out the shortest path using Dijkstra's algorithm.
[Note: You are not required to show any intermediate steps for the above questions. Please write only the final answer. End of Note]
10. Complete the $C$ function findSpiral() given below, which finds the spiral order traversal of a tree.

For the below tree, this function should return 1, 2, 3, 4, 5, 6, 7. Basically, the order of printing of nodes is reversed for each alternate level.


The function findSpiral() takes root node as an input parameter and returns the elements in the spiral form of level order traversal as an array. Max Time Complexity: O(N). Max Auxiliary Space: O(N). Use appropriate data structures to optimize your code. Assume driver function is available to you.

```
/* A binary tree node has data, pointer to left child
    and a pointer to right child
struct Node
{
    int data;
    struct Node* left;
    struct Node* right;
}; */
//Function returns an array containing the level order traversal in spiral form.
int * findSpiral(Node *root)
{
    //Your code here
}```

