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Birla Institute of Technology & Science, Pilani

First Semester 2017-2018

Comprehensive Examination

Marks Obtained: →	
Examiner's Signature: →	

Course:	CS F222
Date and Time:	Dec 11, 2017 FN (09 AM - 12 Noon)

Course Title:	Discrete Structures for Computer Science	<b>AX</b>
Max. Marks, Time, & Set:	Part A 30 (15%) 60 Min. [Closed Book] Part B 60 (30%) 120 Min. [Closed Book]	

Note: In this part (Part A) there are 30 questions each carrying one mark. To answer a question write (in the grid given below) exactly two choices among the four (A, B, C, D) given. To fetch marks both the choices must be correct. Do not write anything in the row below the row labeled 'Choice2'. For overwritten answers one cannot apply for recheck. There is no negative marking. One can start attempting Part-B after submitting Part-A latest by end of time permitted for the Part-A.

Recheck Request ↓	Question No. ↓	Examiner's Comments
Totaling mistake	Total should be ____.	
Please Recheck Qs		

**For Each Question Write (in BLOCK CAPITAL LETTERS) Exactly Two Choices**

Q	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Choice1																														
Choice2																														

**Q01.** True statements for sets A, B and C

- A. If  $A \subseteq B$  and  $B \subset C$ , then  $A \subseteq C$
- B. If  $A \subseteq B$  and  $A \subset C$  then  $B \subseteq C$
- C. If  $A \subseteq B$  and  $B \subseteq C$ , then  $A \subseteq C$
- D. If  $A \subseteq B$  and  $B \subset C$ , then  $A \subset C$

**Q02.** Which are examples of equivalence relations?

- A.  $R = \{(x,y) \mid x,y \in \mathbb{N} \text{ and } x+y \text{ is prime}\}$
- B.  $R = \{(x,y) \mid x,y \in \mathbb{N} \text{ and } x+y \text{ is even}\}$
- C.  $R = \{(x,y) \mid x,y \in \mathbb{N} \text{ and } x+y \text{ is odd}\}$
- D.  $R = \{(x,y) \mid x,y \in \mathbb{N} \text{ and } x+y > 1\}$

**Q03.** If A is the set of all the natural numbers between 6 and 36 and divisible by 8, then true statements:

- A. We can define  $2^{16}$  different relations over A
- B. We can define  $2^8$  different relations over A
- C. We can define 8 different reflexive relations over A
- D. We can define 4096 different reflexive relations over A

**Q04.** Which relations on set  $\{1, 2, 3, 4\}$  are not transitive?

- A.  $R = \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$
- B.  $R = \{(1, 3), (3, 2), (2, 1)\}$
- C.  $R = \{(2, 4), (4, 3), (2, 3), (4, 1)\}$

D.  $R = \{(2, 4), (2, 3), (4, 4), (3,2), (4,2)\}$

**Q05.** Which of the following are partitions of the set  $\{u, m, b, r, o, c, k, s\}$ ?

- A.  $\{\{m, o, c, k\}, \{r, u, b, s, u\}\}$
- B.  $\{\{b, r, o, c, k\}, \{m, u, s, t\}\}$
- C.  $\{\{u, m, b\}, \{r, o, c, k, s\}, \emptyset\}$
- D.  $\{\{b, o, o, k\}, \{r, u, m\}, \{c, s\}\}$

**Q06.** Let R and S be relations from A to B,  $A_1$  and  $A_2$  are subsets of A and  $R(x)$  denotes 'R-relative set of x'. Which statements are true?

- A.  $R(A_1 \cup A_2) = R(A_1) \cap R(A_2)$
- B. If  $A_1 \subseteq A_2$ , then  $R(A_1) \subseteq R(A_2)$
- C. If  $R(a) = S(a)$  for all  $a$  in A, then  $R = S$
- D.  $R(A_1 \cap A_2) \not\subseteq R(A_1) \cap R(A_2)$

**Q07.** The relation  $R = \{(a,a), (b,b), (c,d), (b,d), (a,d)\}$  on set  $\{a,b,c,d\}$  is.

- A. Asymmetric
- B. reflexive
- C. antisymmetric
- D. non-transitive

**Q08.** Which are the examples of anti-symmetric relations?

- A.  $R = \{(a,b) \mid a < b\}$
- B.  $R = \{(a,b) \mid a > b\}$

C.  $R = \{(a,b) \mid a = |b|\}$

D.  $R = \{(a,b) \mid a \geq b\}$

**Q09.** If R is a relation on A, and B is a \_\_\_ of A, the restriction of R to B is \_\_\_\_.

- A. Superset
- B. Subset
- C.  $R \cap (B \times B)$
- D.  $R \cap (A \times A)$

**Q10.** What is the coefficient of  $X^{10}$  in  $(1-X)^{-5}$  and  $(X^3 + X^4 + X^5 + \dots)^2$ ?

- A. 66
- B. 8
- C.  ${}^{14}C_{10}$
- D. 5

**Q11.** The generating functions for the sequences  ${}^k C_n$  and  $a^n$  are:

- A.  $(1 + X)^k$
- B.  $1/(1 - aX)$
- C.  $1/(1 + aX)$
- D.  $1/(1 - X)$

**Q12.** Solving  $a_n - 5a_{n-1} + 6a_{n-2} = 0$  where  $a_0 = 2$  and  $a_1 = 5$ , and given that  $C_1$  and  $C_2$  are constants we get:

- A.  $C_1 = 0$
- B.  $C_2 = 1$

C.  $a_n = 2^n + 3^n$

D.  $a_n = 2^n - 3^n$

**Q13.** If graph  $G_1 = (V_1, E_1)$  is sub-graph of a graph  $G = (V, E)$  then following must hold true:

A.  $V_1 \subseteq V$

B.  $E_1 \subseteq E$

C.  $E_1 \subseteq E \cap (V_1 \times V_1)$

D.  $V_1 \subset V$

**Q14.** Two graphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are isomorphic if there exists a one-to-one \_\_\_\_ function  $f: V_1 \rightarrow V_2$  that preserves \_\_\_\_.

A. Into

B. Onto

C. Reacheability

D. Adjacency

**Q15.** If  $D = \{(a,a) \mid a \in A\}$  is the identity relation on A, which statements are true about a relation R on A?

A. R is reflexive if  $D \subseteq R$

B. R is reflexive if  $D = R$

C. R is irreflexive if  $D \cap R = R$

D. R is irreflexive if  $D \cap R = \emptyset$

**Q16.** If every pair of elements of set A are comparable, A is a \_\_\_\_ or a \_\_\_\_.

A. Poset

B. Well order

C. Totally ordered set

D. Chain

**Q17.**  $R \circ R^{-1}$  is \_\_\_\_ and \_\_\_\_ if R is a reflexive relation on a set A, where  $\circ$  denotes composition.

A. Reflexive

B. Anti-symmetric

C. Symmetric

D. Irreflexive

**Q18.** If R is a reflexive and transitive relation on a set A, then which statements are true?

A.  $R^2$  is reflexive and transitive

B.  $R \cup R^2$  is symmetric

C.  $R^{-3}$  is reflexive and transitive

D.  $R^5$  is reflexive and transitive

**Q19.** Let R and S are relations on set A, then \_\_\_\_  $\subseteq$  \_\_\_\_ for each positive integer.

A.  $(R \cup S)^n$

B.  $(R \cap S)^n$

C.  $S^n \cup R^n$

D.  $R^n \cap S^n$

**Q20.** No simple undirected graph can have degree sequence as:

A. (2,2,3,3).

B. (1,3,3,3,5,6,6).

C. (1,1,3,3,3,4,6,7).

D. (3,3,4,4,4).

**Q21.** For the undirected graph  $G = (\{a, b, c, d, e, f, g, h, i, j\}, \{(a,b), (a,d), (b,c), (b,d), (b,f), (c,d), (c,e), (d,j), (e,f), (f,g), (f,j), (g,h), (g,j), (h,i), (i,i), (i,j)\})$ , which vertices have degree more than three?

A. c

B. f

C. g

D. i

**Q22.** In a simple connected graph  $G = (V, E)$ , there is a vertex of degree at least \_\_\_\_ and a vertex of degree at most \_\_\_\_.

A.  $\lceil 2|E|/|V| \rceil$

B.  $\lfloor 2|E|/|V| \rfloor$

C.  $\lfloor |E|/(2|V|) \rfloor$

D.  $\lceil |V|/2 \rceil$

**Q23.** True about the binary tree representation of the algebraic expression  $(a + 5) \times [(3b + c) \div (d + 2)]$ :

A. There are seven leaf nodes.

B. There are seven internal nodes.

C. There are six internal nodes.

D. The root node has value  $\div$

**Q24.** A simple graph with \_\_\_\_ vertices of \_\_\_\_ degree and no vertices of degree zero must contain a circuit.

A. At least two

B. even

C. no

D. odd

**Q25.** True about steps of Kruskal's Algorithm for finding a minimum spanning tree:

A. In second step any remaining edge of graph is selected if it does not form a circuit with already selected edges.

B. In the first step the edge of minimum value is selected if it does not form a loop.

C. The second step may be repeated.

D. The third step is not a logical check.

**Q26.** True statements are:

A. A connected graph G is a tree iff G has fewer edges than vertices.

B. If G is a connected graph then  $|E| \geq |V| - 1$

C. If a simple connected graph G has only one spanning tree then G is a complete graph.

D. A tree is not a bipartite graph

**Q27.** Number of vertices in a complete directed tree of degree 5 and height 4 ranges in [ \_\_\_\_, \_\_\_\_ ].

A. 125

B. 625

C. 157

D. 781

**Q28.** True statements are:

A. The complete tripartite graph  $K_{1,2,3}$  is non-planar

B. Any graph with 5 or fewer vertices is planar

C. Critical planar graph may not be connected

D.  $K_{2,2}$  is planar.

**Q29.** Distributive property is a requirement for:

A. Ring

B. Field

C. Monoid

D. Group

**Q30.** Using how many colors we can color vertices of a planar graph such that no two adjacent vertices are colored in same color?

A. 2

B. 3

C. 4

D. 5

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**Q30.** Which of the following are partitions of the set  $\{u, m, b, r, o, c, k, s\}$ ?

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- B.  $\{\{b, r, o, c, k\}, \{m, u, s, t\}\}$
- C.  $\{\{u, m, b\}, \{r, o, c, k, s\}, \emptyset\}$
- D.  $\{\{b, o, o, k\}, \{r, u, m\}, \{c, s\}\}$

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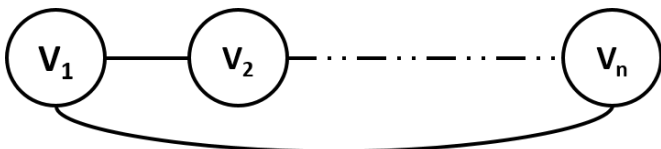
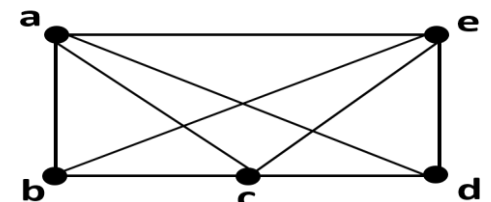
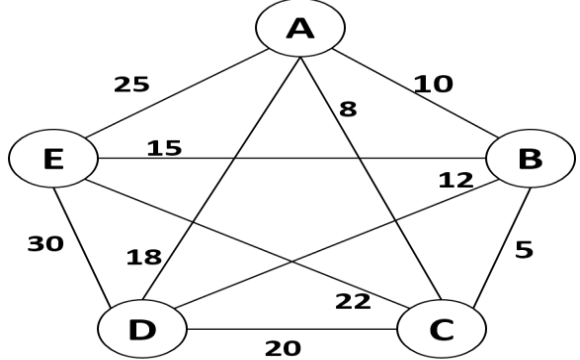
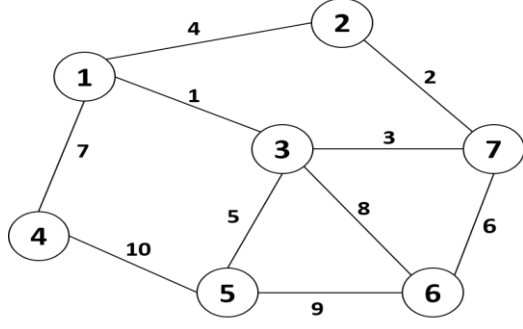
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**Note: In this part (Part B) there are 16 questions. Q1 to Q15 each carries 3 marks. Q16 has 15 parts and each part carries one mark. To answer a question, write the answer in the cell across the question. For overwritten answers one cannot apply for recheck. There is no negative marking.**

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Q#.	Question (Each question carries three marks.). Figures are given Overleaf.	Answer ↓
01.	An intersection graph is an undirected graph whose edges are specified by the rule that there is an edge between vertices A and B iff $A \neq B$ and $A \cap B \neq \Phi$ . What is the chromatic number of the graph with vertex set $V = \{\{1,2,3\}, \{1,9,10\}, \{2,4,6,8,10\}, \{3,4,5\}, \{5,6,7\}, \{7,8,9\}\}$ ?	
02.	What is the greatest element (if any), of the set of integers $\geq 0$ under the divisibility relation i.e. $aRb$ implies a divides b?	
03.	What are the minimal elements (if any), of the set, $N$ , of all natural numbers $\geq 2$ under divisibility relation i.e. $aRb$ implies a divides b? Please note that $\infty$ is not a number.	
04.	Consider the binary relation: $S = \{(x, y) \mid y = x + 1 \text{ where } x, y \in \{0, 1, 2, \dots\}\}$ . The reflexive transitive closure (first perform transitive then reflexive) of $S$ is :	
05.	A bi-partite graph cannot be planar if the degree of all nodes in the graph $G(V,E)$ is greater than or equal to $n$ . What is the minimum value of $n$ .	
06.	The complement of a graph $G$ is a graph $H$ on the same vertices such that two distinct vertices of $H$ are adjacent if and only if they are not adjacent in $G$ . If $G$ is a simple graph with 15 edges and $H$ (the complement graph of $G$ ) has 13 edges, how many vertices does it have (Give numeric value)?	
07.	Let $G$ (given in <b>Fig. 1</b> ) be an undirected graph on $n$ ( $n > 2$ ) nodes labeled $v_1, v_2 \dots \dots v_n$ . What is the chromatic number of $G$ ? The chromatic number of a graph is the smallest number of colors needed to color the vertices of so that no two adjacent vertices share the same color.	
08.	Let $Z_n$ be the set $\{0, 1, 2, \dots, n - 1\}$ . The binary operation $\otimes$ , on this set is defined as $a \otimes b$ which is equal to the unique integer in $Z_n$ that is congruent (mod $n$ ) to the usual product $ab$ i.e. $a \otimes b = ab \text{ mod } n$ . Then, how many numbers would never have an inverse in $(Z_6, \otimes)$ ?	
09.	The number of regions in the plane, determined by a planar representation of the graph given in <b>Fig. 2</b> .	
10.	In a party there are 25 people, each of a distinct age ( $\geq 1$ ) and not more than 25 years old. A group of people will have lunch together only when the ages of any two people in the group are relatively prime. What is the size of the largest group that can lunch together?	
11.	How many different total number of connected undirected simple graphs without self-loops are possible on the 6 labeled vertices $a_i, 1 \leq i \leq 6$ , such that degree of the vertices $a_1, a_3$ , and $a_5$ is one?	
12.	While using Kruskal's algorithm to find the minimum spanning tree for the weighted graph shown in <b>Fig. 3</b> , what are the first three edges to be chosen?	
13.	Consider the graph given in <b>Fig. 4</b> . Which edges, and in which order, are selected by Prim's algorithm if it starts at vertex 1?	
14.	Tuple $(x,y,i)$ denotes transition from state $x$ to $y$ under the input $i$ . Consider a finite state automata with transitions $\{(a,a,0), (b,a,0), (a,b,1), (b,c,1), (c,a,0), (c,a,1), (d,b,1), (d,c,0)\}$ , for initial state $d$ and final states in $\{a\}$ . Find the total ways the state $a$ can be reached from a state other than state $a$ .	
15.	The particular solution of the recurrence relation $a_{r+2} - 4a_r = r^2 + r - 1$ , is given by ____.	

Q16.	Question (Each question carries one mark.)	Answer ↓
16.01	Consider the algebraic system $(Q, \oplus)$ , where $a \oplus b = a * b/2$ , where $Q$ is the set of rational numbers and $*$ is usual multiplication. What is identity element of this algebraic system?	
16.02	The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ is a group under multiplication modulo 15. The inverses of 4 and 7 are respectively.	
16.03	If a connected planar simple graph $G$ has 7 vertices with degrees 1,1,1,1,2,3,3 then the number of regions in a planar representation is:	
16.04	Let $G(V,E)$ be the non-planar graph with the minimum possible number of edges i.e no non-planar graph exist with edges less than $ E $ . What is the value of $ E $ .	
16.05	The inclusion of which set(s) into $S = \{\{1,2\}, \{1,2,3\}, \{1,3,5\}, \{1,2,4\}, \{1,2,3,4,5\}\}$ is necessary and sufficient to make $S$ a complete lattice under the partial order defined by set containment?	
16.06	Which of these symbols $\Theta$ $O$ $\Omega$ $o$ $\omega$ can go in the blank space? (List all that apply.) $2n + \log n$ is _____ (n)	
16.07	Which of these symbols $\Theta$ $O$ $\Omega$ $o$ $\omega$ can go in the blank space? (List all that apply.) $\log n$ is _____ (n)	
16.08	Which of these symbols $\Theta$ $O$ $\Omega$ $o$ $\omega$ can go in the blank space? (List all that apply.) $n2^n$ is _____ (n)	
16.09	Which of these symbols $\Theta$ $O$ $\Omega$ $o$ $\omega$ can go in the blank space? (List all that apply.) $n^2$ is _____ ( $n^2$ )	
16.10	If sets $S$ and $T$ have $n$ elements in common, then $S \times T$ and $T \times S$ have ___ elements in common.	
16.11	Let $R = \{(4, 4), (4, 10), (6, 6), (6, 8), (8, 10)\}$ is relation defined on set $\{4, 6, 8, 10\}$ . By including which tuple or tuples (excluding which are already in $R$ ) in $R$ we can get transitive closure of $R$ .	
16.12	In a graph matching a vertex can have at ___ degree one.	
16.13	A connected graph $G$ is Eulerian if and only if every vertex has _____ degree.	
16.14	If $G$ is a connected simple graph with $n \geq 3$ vertices and if the degree $deg(v) \geq$ ___ for every vertex of $G$ , then $G$ is Hamiltonian.	
16.15	In contrast to Finite State Machines with output, Finite state automata do not produce output but they do have a set of _____ states.	

	
Fig. 1 ↑	Fig. 2 ↑
	
Fig. 3 ↑	Fig. 4 ↑

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Birla Institute of Technology & Science, Pilani  
 First Semester 2017-2018  
 Comprehensive Examination

Marks  
 Obtained: →

Examiner's  
 Signature: →

<b>Course:</b>	CS F222
<b>Date and Time:</b>	Dec 11, 2017 FN (09 AM - 12 Noon)

<b>Course Title:</b>	Discrete Structures for Computer Science	
<b>Max. Marks, Time, &amp; Set:</b>	Part A 30 (15%) 60 Min. [Closed Book] Part B 60 (30%) 120 Min. [Closed Book]	<b>BY</b>

Note: In this part (Part B) there are 16 questions. Q1 to Q15 each carries 3 marks. Q16 has 15 parts and each part carries one mark. To answer a question, write the answer in the cell across the question. For overwritten answers one cannot apply for recheck. There is no negative marking.

<b>Recheck Request ↓</b>	<b>Question No. ↓</b>	<b>Examiner's Comments</b>
Totaling mistake	Total should be ____.	
Please Recheck Qs		

Q#.	Question (Each question carries three marks.). Figures are given Overleaf.	Answer ↓
01.	The complement of a graph $G$ is a graph $H$ on the same vertices such that two distinct vertices of $H$ are adjacent if and only if they are not adjacent in $G$ . If $G$ is a simple graph with 15 edges and $H$ (the complement graph of $G$ ) has 13 edges, how many vertices does it have (Give numeric value)?	
02.	Let $G$ (given in <b>Fig. 1</b> ) be an undirected graph on $n$ ( $n > 2$ ) nodes labeled $v_1, v_2, \dots, v_n$ . What is the chromatic number of $G$ ? The chromatic number of a graph is the smallest number of colors needed to color the vertices of so that no two adjacent vertices share the same color.	
03.	Let $Z_n$ be the set $\{0, 1, 2, \dots, n - 1\}$ . The binary operation $\otimes$ , on this set is defined as $a \otimes b$ which is equal to the unique integer in $Z_n$ that is congruent (mod $n$ ) to the usual product $ab$ i.e. $a \otimes b = ab \pmod n$ . Then, how many numbers would never have an inverse in $(Z_6, \otimes)$ ?	
04.	The number of regions in the plane, determined by a planar representation of the graph given in <b>Fig. 2</b> .	
05.	In a party there are 25 people, each of a distinct age ( $\geq 1$ ) and not more than 25 years old. A group of people will have lunch together only when the ages of any two people in the group are relatively prime. What is the size of the largest group that can lunch together?	
06.	How many different total number of connected undirected simple graphs without self-loops are possible on the 6 labeled vertices $a_i, 1 \leq i \leq 6$ , such that degree of the vertices $a_1, a_3$ , and $a_5$ is one?	
07.	While using Kruskal's algorithm to find the minimum spanning tree for the weighted graph shown in <b>Fig. 3</b> , what are the first three edges to be chosen?	
08.	Consider the graph given in <b>Fig. 4</b> . Which edges, and in which order, are selected by Prim's algorithm if it starts at vertex 1?	
09.	Tuple $(x, y, i)$ denotes transition from state $x$ to $y$ under the input $i$ . Consider a finite state automata with transitions $\{(a, a, 0), (b, a, 0), (a, b, 1), (b, c, 1), (c, a, 0), (c, a, 1), (d, b, 1), (d, c, 0)\}$ , for initial state $d$ and final states in $\{a\}$ . Find the total ways the state $a$ can be reached from a state other than state $a$ .	
10.	The particular solution of the recurrence relation $a_{r+2} - 4a_r = r^2 + r - 1$ , is given by ____.	
11.	An intersection graph is an undirected graph whose edges are specified by the rule that there is an edge between vertices $A$ and $B$ iff $A \neq B$ and $A \cap B \neq \Phi$ . What is the chromatic number of the graph with vertex set $V = \{\{1,2,3\}, \{1,9,10\}, \{2,4,6,8,10\}, \{3,4,5\}, \{5,6,7\}, \{7,8,9\}\}$ ?	
12.	What is the greatest element (if any), of the set of integers $\geq 0$ under the divisibility relation i.e. $aRb$ implies $a$ divides $b$ ?	
13.	What are the minimal elements (if any), of the set, $N$ , of all natural numbers $\geq 2$ under divisibility relation i.e. $aRb$ implies $a$ divides $b$ ? Please note that $\infty$ is not a number.	
14.	Consider the binary relation: $S = \{(x, y) \mid y = x + 1 \text{ where } x, y \in \{0, 1, 2, \dots\}\}$ . The reflexive transitive closure (first perform transitive then reflexive) of $S$ is :	
15.	A bi-partite graph cannot be planar if the degree of all nodes in the graph $G(V, E)$ is greater than or equal to $n$ . What is the minimum value of $n$ .	

Q16.	Question (Each question carries one mark.)	Answer ↓
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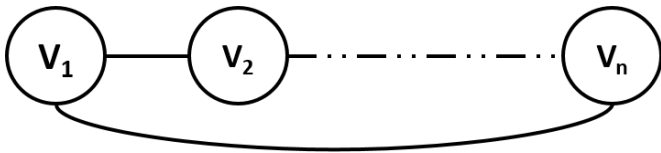


Fig. 1 ↑

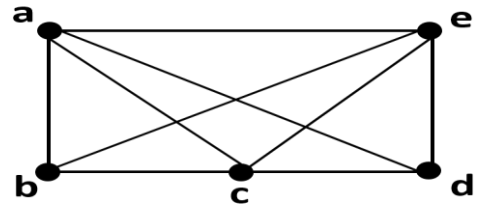


Fig. 2 ↑

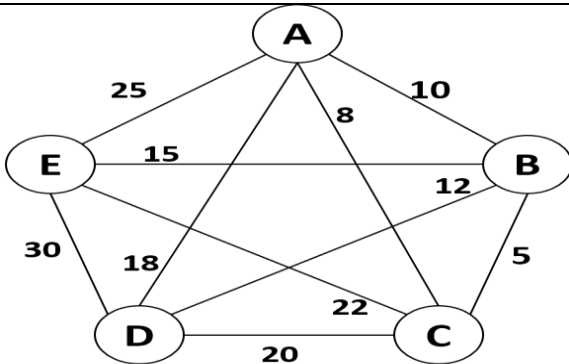


Fig. 3 ↑

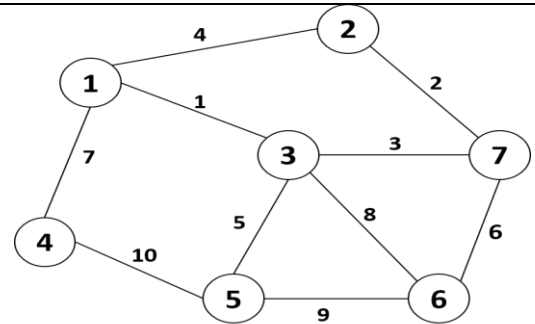


Fig. 4 ↑