BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
FIRST SEMESTER 2017-2018
CS F222 - Discrete Structures for Computer Science

Oct 13, 2017
09:00 AM – 10:30 AM

Mid Semester Test (Open book)

Maximum Marks 70
(35% Weight)

Instructions: The question paper has two parts, namely Part A and Part B each 35 marks. Both parts are open book. Each part is to be attempted in separate answer booklet. Student can start attempting Part B, only after submitting Part A. Students should write corresponding parts as PART-A or PART-B, in the top right corner of the cover page of the answer booklet.

Part A (Maximum time permitted: 45 min.)

Q1. Write precise and concise answers for the following:

Q1.1 Let $\mathcal{H} = \{1,2,3,4,5,6,7\}$ and $\mathcal{K} = \{1,2,3,4\}$ and let $\mathcal{P}$ be the power set of $\mathcal{H}$ and let $\mathcal{R}$ be the relation on $\mathcal{P}$ defined as, $\mathcal{R} = \{\, (X,Y) \mid \; |X \cap K| = |Y \cap K| \}$. How many elements does the equivalence class of $\{1,2\}$ have? **[3]**

Q1.2 Consider the relation R on the set of all functions $f : Z \to Z$, defined as $\{\, (f, g\,) \mid f(0) = g(0) \vee f(1) = g(1)\}$. Is R an equivalence relation? Justify your answer. **[3]**

Q1.3 **False Claim:** Suppose R is a relation on A. If R is symmetric and transitive, then R is reflexive. Give a counter-example of a relation on the set A:={a,b,c} to contradict the claim. Also find the flaw in the following proof for the claim. **[4]**

False proof. Let x be an arbitrary element of A. Let y be any element of A such that xRy. Since R is symmetric, it follows that yRx Then since xRy and yRx, we conclude by transitivity that xRx. Since x was arbitrary, we have shown that $\forall x \in A$, (xRx), so R is reflexive. Also, provide a minor fixation to the claim, to make it correct.

Q2. What is the set of all numbers which satisfy the equation $3x + 5y$, where $x, y \in \{0,1,2,3\ \ldots\ldots\}$? Prove your result by using <u>Strong Induction</u>. Remember to formally identify the induction hypothesis $P(n)$. Establish the base case. Prove that $P(n) \Rightarrow P(n+1)$. Conclude that $P(n)$ holds for all $n \geq k$, where k is some constant. **[4]**

Q3. Prove the following:

Q3.1 Let $\hat{v}$ be the set of vertices adjacent to the vertex v in a graph, i.e. $\hat{v} = \{v' \mid (v, v')\ is\ an\ edge\ of\ the\ graph\}$. Suppose, f is an isomorphism from graph G to graph H. Prove that $f(\hat{v}) = \widehat{f(v)}$, where $\widehat{f(v)}$ is the set of vertices adjacent to the vertex $f(v)$. **[5]**

Q3.2 Show that a simple graph G with n vertices is connected if it has more than $\left(\frac{1}{2}\,(n-1)(n-2)\right)$ edges. **[5]**

Q4. Derive the expression for the generating function $G(x)$, corresponding to the sequence which gives the number of ways n unlabeled balls can be distributed into k distinct bins, where each bin may contain a maximum of m =< n balls. **[4]**

Q5. Use the Well-ordering Principle to prove that there is no solution over the non-zero positive integers to the equation, $4a^3 + 2b^3 = c^3$, where a, b, and c are non-zero integers to be determined such that they satisfy the given equation. **[7]**

Part B (To be attempted after submission of part A)

Q1. For relation R on a set A = {a,b,c,d}, give the transitive closure (M_{R^*}) of R in the following situations: **[2*4=8]**

a) Every node is reachable from every other node

b) There is an exactly one node which cannot be reached from any node

c) There is a node which can be reached from all other nodes, but once you reach this node, you are stuck.

d) Starting from any node, you can reach all other nodes, but you cannot come back to the starting node once you leave that node.

Q2. Two persons A and B gamble dollars on the toss of a fair coin. A has \$70 and B has \$30. In each play, either A wins \$1 from B or loses \$1 to B. The game is played without stopping until one wins all the money of the other or the game goes on forever. Find the probabilities of the following three possibilities, using the concept of recursion: **[4+3=7]**

a) A wins all the money of B.

b) A loses all his money to B.

c) The game continues forever.

Q3. Suppose N persons are part of a social media site. A relation captures FOLLOWS, i.e., there is a directed edge between person A_i to person A_j if A_i is a follower of A_j. A sample relation for 5 persons $\{A_1, A_2, A_3, A_4, A_5\}$ is given on right hand side: **[6]**

$$R = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find out the followers of followers of followers for each person in the given relation R. Show all steps.

Q4. Answer the following:

Q4.1 Give a recursive definition of the set of bit strings that have the same number of zeros and ones. **[4]**

Q4.2 Using the concept of generating functions determine the number of different ways in which 10 identical chocolates can be given to four children if each child receives at least two chocolates. (Note: The children are distinguishable) **[4]**

Q5. How many isomorphic mappings are possible from G to H? Enumerate all of them. Justify your answer. **[6]**