BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (RAJ.)

First Semester 2023-24

CS F222 DISCRETE STRUCTURES IN COMPUTER SCIENCE

Name:______________________________ ID:________________________________

Dec 13, 2023 **COMPREHENSIVE EXAMINATION** 80M, 180 Min

Recheck Request:

Instructions:

i. Please write your name and ID on this sheet at the top.
ii. This paper consists of two parts: Part A is in-built, closed book, and Part B is open-book.
iii. Please write your answers precisely, with appropriate justifications.
iv. Please write legibly. Neatly written paper may fetch bonus credit, and poorly written may fetch negative credit
v. For both Part A and Part B, you may write your assumption(s), if required.
vi. Part A is to be submitted no earlier than 10.45 AM and no later than 11.15 AM.

PART A

1. An oil company at its refinery has seven major buildings that are connected by underground tunnels, as shown in the diagram below. Because of the possibility of a major explosion, there is a need to reinforce some of these tunnels to avoid a potential cave-in. The company wants to be able to go from any building to any other in case of a major fire aboveground, but it wants to avoid reinforcing more tunnels than necessary. How can this be done? [2]

2. Is it possible to have a connected weighted graph in which the same edge is part of every minimal spanning tree and every maximal spanning tree? Why, or why not? [2]

3. If the vertices of K_n are labeled as 1,2,3,4,… and depth-first search is applied to K_n, how many different depth-first search numberings can be there? [1]

4. Obtain a depth-first search numbering for the vertices of the graph shown below. If there's a choice of vertices, choose the vertex that appears first in alphabetical order. [2]

5. Use the depth-first search numbering obtained above in Q4 to draw a spanning tree of the graph. [1]

6. Can $K_{m,n}$ have an Euler circuit? If not why? If yes, under what conditions? [1.5]

7. Can $K_{m,n}$ have an Hamiltonian cycle? If not why? If yes, under what conditions? [1.5]

8. Find the components of the following graphs. [1.5]

9. What is the upper bound on the number of edges a bipartite graph with n vertices can have? Give complete justification. [4]

10. The number of non-isomorphic simple graphs with six vertices and four edges is _________. [2]

11. Depth first search and breadth first search produce identical spanning trees for graphs which ______________________________________. [1.5]

12. A connected component of a given graph is a ____________________________________. [1.5]

13. For the following graph, determine the number of paths of lengths 1, 2, 3, and 4 from V_1 to V_1, and from V_4 to V_3. [3]

14. Devise an algorithm for finding the second shortest spanning tree in a connected weighted graph. [3]

15. Your senior is writing a compiler for a programming language named *cradle* and has forgotten the concepts learnt in Discrete Structures. How would you help him/her by using the concepts you have learnt in this course. Give one instance, with not more than one or two sentences. [1]

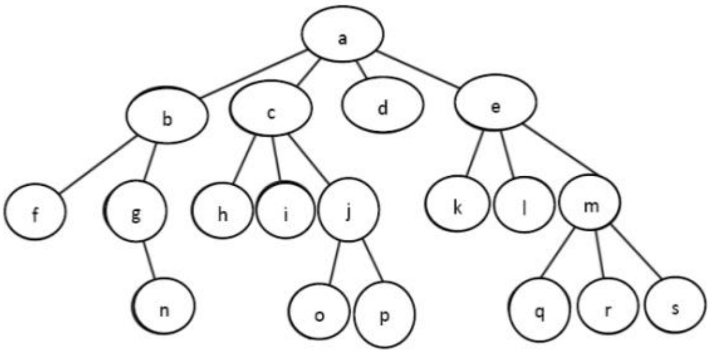
16. Prove that there are infinitely many solutions in positive integers to a, b and c to the equation $a^2 + b^2 = c^2$. [3]

17. Let G be a connected graph and T be a spanning tree of G. An edge of G which is in T is called a branch of T. An edge of G which is not in T is called a chord of T. (a). Prove that any edge of a connected graph G is a branch of some spanning tree of G. (b). Prove or disprove the statement *Any edge of a connected graph G is a chord of some spanning tree of G* . [3]

18. In a connected undirected graph G, a vertex v is called the center if the maximum distance between v and any other vertex is as small as possible. The distance between two vertices is the minimum number of edges in a path between them. [3]
    a. Draw a connected graph G_1 that has one center and eleven vertices and the degree of each vertex in G is two or more.

b. Draw a connected graph G_2 that has two or more centers.

19. Find the prerorder and the postorder traversal of the tree shown below. [2]



Prerorder :

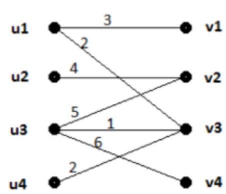
Postorder :

20. Evaluate the below expressions showing all the steps clearly. [2]

a. + * 2 * * * + 3 * + 2 1 3 4 2 2 2

b. 2 4 3 2 + 2 * + 3 * 1 * 2 * * 7 +

21. Find a perfect matching M of the below given graph such that the cumulative weight of the edges in M is the smallest possible. [1M]

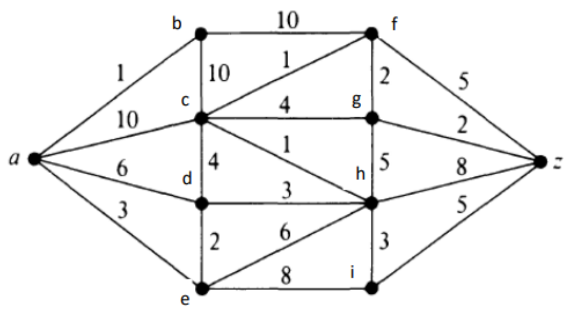


22. Draw a graph G with the maximum degree 4 and the chromatic index is 7. [1M]

23. Is it possible to tile a 6 x 6 checker board using dominoes in such a way that each ruling is crossed by at least one domino? If your answer is "yes" draw such a tiling; otherwise, justify your answer. [3M]

24. Show that in a connected simple planar graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges. [2]

25. Apply the greedy coloring algorithm to the edges of the graph shown in Figure 2 in the increasing order of their weights. The set of colors is: {c1, c2, c3, . . .}. Write the edges and their respective colors clearly. In case of multiple edges with same weights, use the ordering left over right and bottom over top in this order. For example, for edges with weights, the ordering would be edge *ac* followed by *bc* and then by *bf*. [4]



26. Write the expression for the recursive function given below inside the box. [2]

$p(n,a) = a^2$ if n=1

$p(n,a) = p(n-1,a)^2$ otherwise

27. Let A = {a,b,c,d}. A relation R on set A is defined as
R = {(a,a), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,d) } Which of the following is/are true? [1.5]

   a. R is an equivalence relation.
   b. R is irreflexive or asymmetric relation.
   c. R is transitive.
   d. R is symmetric or asymmetric relation.

28. Determine if the poset with Hasse diagram shown below is a lattice. Justify. [2]

29. Complete the following proof to show that every well-formed formulae for compound propositions contains an equal number of left and right parentheses. [3]

<u>Basis step</u>: Each of the formulae T, F, and s contains no parentheses so clearly they contain an equal number of left or right parentheses.

<u>Recursive step</u>: Assume p and q are well-formed formulae each containing an equal number of left and right parentheses. That is, if l_p and l_q are the number of left and right parentheses in p and q, and r_p and r_q are the number of right parentheses in p and q respectively, then $l_p=r_p$ and $l_q=r_q$.

Write your answer for Q8 here in the box below

Solution for Q8

ROUGH