BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (RAJ.)

First Semester 2023-24

CS F222 DISCRETE STRUCTURES IN COMPUTER SCIENCE

Oct 11, 2023 **MID-SEMESTER EXAMINATION** 50M, 90 Min

1. Write a single statement describing the output of the following recursive procedure. **[2]**

```
int fun(int x, int y)
{
  if (x == 0)
    return y;
  return fun(x - 1, x + y);
}
```

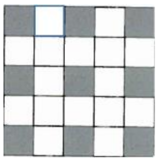

Fig 1

2. Write a recursive definition for the set of bit strings $S = \{0^r1^s \mid r >= 0, s >= 0\}$. **[3]**
3. Is $7^m - 1$ divisible by 6 for all positive integers m? Why, or why not? **[3]**
4. Prove that if n is a positive integer and 2^n-1 is prime, then n is prime. Use proof by contradiction. **[3]**
5. Consider the set S = {a, b}. Draw the directed graph representation of each relation that can be defined on set S. Which of these relations are transitive? **[3]**
6. Is it possible to define a relation which is neither symmetric nor anti-symmetric? If not, why not? If yes, how many such relations are possible on a set with n elements. **[5]**
7. Let f(n), n >= 0, denote the Fibonacci sequence. Prove by induction that 1 < (f(n+1)/f(n)) < 2 for all n>=3. Assume f_0=0. **[6]**
8. Consider the relation R_n as discussed in the class. **[6]**
   a) Define R_n.
   b) Prove that R_n is an equivalent class.
   c) Write down the sets resulting from the partition arising from the relation R_4, on the set of all bit strings.

9. Consider a 5x5 chequer board as shown in Fig 1. **[7]**
   a. Can we tile a 5x5 checker board obtained by removing a “black” square using right triominoes? Justify your answer by writing a proof.
   b. Can we tile a 5x5 checker board obtained by removing a “white” square using right triominoes? Justify your answer by writing a proof.

10. Let S be a set of bit strings defined recursively by
    Basis Step: λ, 1 ε S
    Recursive Step: If w ε S, then 0w ε S and w0 ε S.
    Prove that S is the set of all bit strings with no more than one 1. **[6]**

11. Consider the relation R defined on A = {1,2,3}, R = {(1,2), (1,3), (2,1), (2,2), (3,1), (3,3)}
    a) Compute a) $(R^{-1})^2$ and b) $\bar{R}^2$.
    b) Compare the time complexities of computing the transitive closure of R by employing connectivity relation and by employing the Warshal’s algorithm. **[6]**

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