Birla Institute of Technology & Science, Pilani (Pilani Campus)
CS F320 Foundations of Data Science
Mid-semester Examination

Date: 10/10/2023 Weightage: 30% Duration: 90 minutes Max. marks: 40

1. a). What is the tailbound of a probability distribution?
   b). What is the drawback in the tail bound given by Markov's inequality?
   c). How Chebyshev inequality gives a tighter or better tailbound than Markov?
   d). State and derive the Law of Large Numbers (LLN) mathematically. [1+1+1+3=6]

2. Let x be a random variable whose p.d.f. is

$$f(x) = \begin{cases} \frac{1}{(ln2)x} & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   Verify Markov's inequality for $a = 1.5$, by finding $P(x \geq 1.5)$ and $\frac{E(x)}{1.5}$. [3]

3. Explain how the bound for the probability i.e. $P(|\bar{x} - E(x)| \geq \epsilon)$ in LLN will change and why for the following cases,
   a). Increasing $\boldsymbol{n}$, when $Var(x)$, and ϵ are constants.
   b). Increasing $\boldsymbol{Var(x)}$, when n, and ϵ are constants.
   c). Increasing $\boldsymbol{\epsilon}$, when n, and $Var(x)$ are constants. [1+1+1=3]

4. Show mathematically that most of the volume of high dimensional objects lies near the surface. [3]

5. Find the singular values of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

   and find the singular value decomposition of A. [2+4=6]

6. a). State three differences between Frequentist and Bayesian approach towards probability.
   b). What will be difference in the parameter estimates given by the Frequentist and Bayesian approach when the number of samples i.e., 'n' is very large? [3+1=4]

7. a). Explain the different steps involved in principal component analysis (PCA), mathematically.
   b). Find the relationship between eigenvalues and variance in the data by solving a constrained optimization problem.
   c). What is a scree plot? [3+2+1=6]

8. Write the mathematical condition for a convex function? Prove that $f(x) = x^2+1$ is a convex function. [1+2=3]

9. Use the Lagrange multiplier method to find the global maximum and minimum values of the function $f(x, y) = 2x + y$ subject to the constraint $x^2 + y^2 = 5$. [3]

10. a). What is a kernel?
    b). What is the use of kernel functions?
    c). Explain the Mercer condition for kernel functions. [1+1+1=3]

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